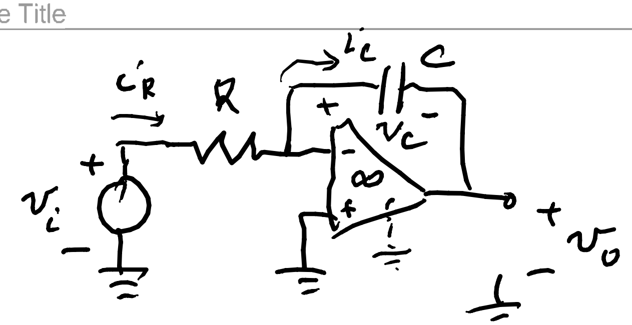
EE 610
09/06/11

Note Title 9/5/2011

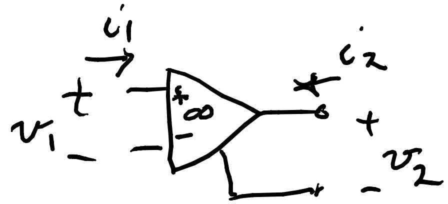


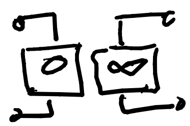
$i_R = v_i/R = i_C = sC \cdot v_C$

$v_0 = -v_C$ by virtual gnd at op-amp input

$\frac{v_i}{R} = sC(-v_0)$

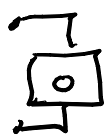
$\frac{v_0}{v_i} = \frac{-1}{RsC}$



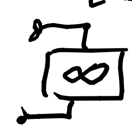


$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$ describes a nullor

input = nullator $v = i = 0$

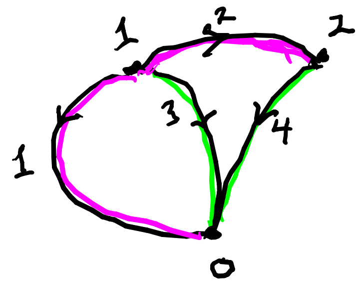


output = norator $\Rightarrow$ v = arbitrary, i = arbitrary

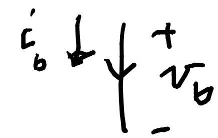


Use KCL, KVL & laws of components

use graph theory:



$i_b = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}_b$, $v_b = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}_b$



tree branches

links branches

v_t = tree branch voltages $= \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_b$ $\quad v_\ell$ = links $= \begin{bmatrix} v_3 \\ v_4 \end{bmatrix}_b$

i_t , i_ℓ

$i_b = \mathcal{J}^T i_\ell$, $v_b = C^T v_t$ $\quad \Leftarrow$ KCL, KVL

Power in from outside $\equiv 0 = v_b^T \cdot i_b = [v_1, v_2, v_3, v_4]_b \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}_b$

$0 = v_b^T \mathcal{J}^T i_\ell$ $\quad$ holds for all i_ℓ

$\Longrightarrow \quad v_b^T \mathcal{J}^T = 0 = [0, \ldots, 0] \Rightarrow \underset{\ell}{0} = \mathcal{J} v_b$

$0 = v_t^T i_b = v_t^T C i_b$ $\quad$ holds for all $v_t \Rightarrow \underset{t}{0} = C i_b$

$\mathcal{C}$ = cut set matrix, $t \times b$ matrix, (2×4 in this ex)

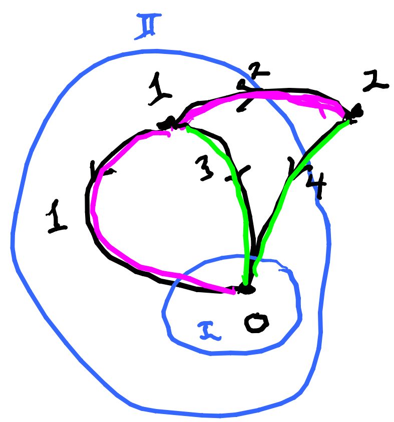
t = # of branches in the tree

$\mathcal{T}$ = tie set matrix, $\ell \times b$ matrix

b = # of branches

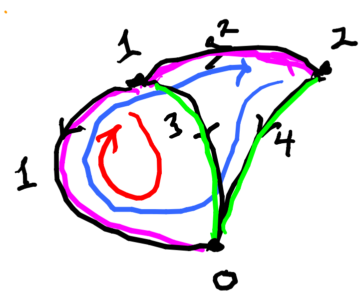
ℓ = # of links

$b = t + \ell$



$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \mathcal{C}\, i_b = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{1_t} & \Big| & \underbrace{\begin{matrix} 1 & 1 \\ 0 & -1 \end{matrix}}_{K_i} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}_b$$

guarantees linearly independent KCL



$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \mathcal{T}\, v_b = \begin{bmatrix} \underbrace{\begin{matrix} -1 & 0 \\ -1 & 1 \end{matrix}}_{K_v} & \Big| & \underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{1_\ell} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}_b$$

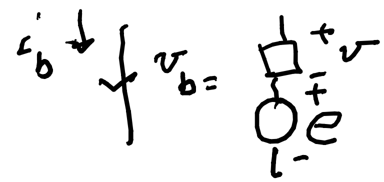
guarantees linearly independent KVL

here $K_v = -K_i^T$

$P = 0 = v_b^T i_b = v_t^T e \mathfrak{J}^T i_l \Rightarrow e \mathfrak{J}^T = 0_{t \times l}$

$$0_{t \times l} = \left[\underline{1}_t \;\vdots\; K_i \right] \begin{bmatrix} K_v^T \\ \underline{1}_l \end{bmatrix} \Rightarrow 0_{t \times l} = \underline{1}_t \cdot K_v^T + K_i \cdot \underline{1}_l \Rightarrow K_i = -K_v^T$$

$Av = Bi$



$\Rightarrow v_b = v + e$

in the example: $e = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} G & 0 & 0 & 0 \\ 0 & sC & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \qquad G = 1/R$$

laws for components