

## EE 610 Final Exam Fall 2011

Take Home. Open Book Open Notes 100 points total  
Due in Room AVW 1364 at the final period along with your notebook. Your signature certifies that the work is your own.

1. (30 points = 6 points per synthesis)

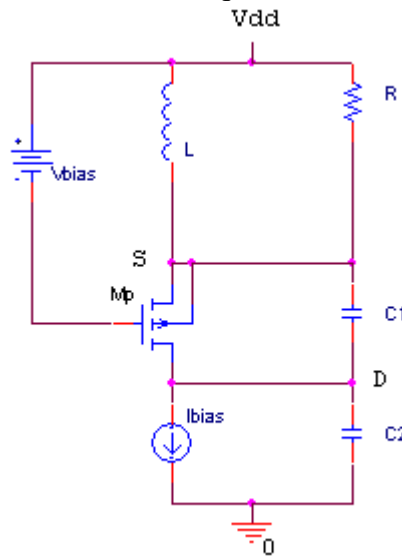
Synthesize  $y(s) = \frac{s(s^2+6)}{s^2+3}$  by 5 methods (1<sup>st</sup> & 2<sup>nd</sup> Foster & Cauer & via Richards'

function sections with  $k=3$ )

2. (30 points)

The following circuit is a Colpitts oscillator for which  $\omega_0^2 = 1/(LC)$  where  $C = (C_1 C_2)/(C_1 + C_2)$ .

- Draw the small signal equivalent low-frequency circuit for the oscillator and from it give the circuit graph (ignore  $M_1$ 's output resistance but include its  $g_m$ ). Orient from nodes S and D to ground (use these branches in a tree) and from S to D. Give the cut-set matrix.
- Use inductor current and voltages on the capacitors and give state variable equations in matrix form ( $A$  is  $3 \times 3$ ). Use  $x = [i_L, v_{C1}, v_{C2}]^T$  as the state.
- From the state variable equations check the oscillation conditions giving the necessary  $g_m$  if  $R < 0$  (hint: equate real and imaginary parts to zero).



3. (40 points; 10 points per part)

Given two positive-real functions  $f(s)$  and  $g(s)$ , possibly not rational in  $s$ , it is known that  $f(g(s))$  is positive-real.

- Give those real constants  $a$  for which  $af(g(s))$  is positive-real.
- Is  $f(s) + g(s)$  always positive-real? Is  $f(s) - g(s)$  always positive-real?
- Is  $f(s)/g(s)$  always positive-real?
- Exhibit a circuit that has a non-rational in  $s$  positive-real input admittance.