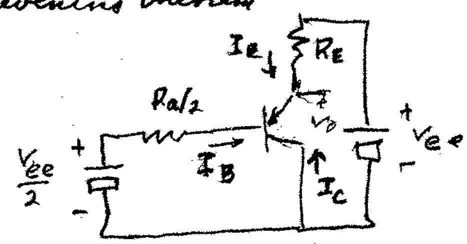
EE303H Midterm Solutions

#1 By Thevenin's theorem



1) $V_{ee} - V_0 = 10 - 8 = 2 = R_E I_E$

2a) $I_E = -\frac{I_C}{\alpha} = -\frac{\beta I_B}{\alpha} = -\frac{\beta}{\beta/(1+\beta)} I_B = -(1+\beta) I_B$

2b) $\Rightarrow I_B = \frac{-1}{(1+\beta)} I_E$

3) KVL: $-\frac{V_{ee}}{2} + \frac{R_a}{2} I_B + (V_{BE}) + V_0 = 0 = -5 + \frac{10^6}{2} \times \left(\frac{-1}{1+99}\right) I_E + (-0.7) + 8 = 0$

3a) $\Rightarrow 3 - 0.7 = 2.3 = \frac{10^6}{2 \times 10^2} I_E = \frac{10^4}{2} I_E$

* 4) $\Rightarrow I_E = 4.6 \times 10^{-4} = 0.46$ milli Amps

From 1)

* $R_E = \frac{2}{I_E} \overset{3a)}{=} \frac{10^4}{2.3} = \frac{10}{2.3} \times 10^3 = 4.35\ k\Omega$

#2. $T(s) = \frac{s(s^2+4)}{(s+3)(s+5)} = \frac{s(s+j2)(s-j2)}{(s+3)(s+5)} \Rightarrow \frac{s^3}{s^2} \Rightarrow \infty$ as $s \to \infty$

zeros: $s = 0, \pm j2$

poles: $s = \infty, -3, -5$

seen by inspection

#3. $I_i = \beta (V_{GS} - V_{TO})^2 (1 + \lambda V_{GS})$

$= 10^{-4} (4-1)^2 (1 + 0.01 \times 4)$

a) $= 10^{-4} (9)(1.04) = 9.36 \times 10^{-4} A = 0.936$ milli A

b) For $I_o = I_i$ need $V_{DS} = V_{GS} = V_o = 4V$

c) as $V_{GS} = 4$, the right transistor is in saturation when $V_{DS} \geq V_{GS} - V_{TO} = 3V$

$\therefore$ for $V_o = V_{DS} = 7$

$I_o = \beta (V_{GS} - V_{TO})^2 (1 + \lambda V_{DS}) = I_i \cdot \left(\frac{1 + \lambda V_{DS}}{1 + \lambda V_{GS}} \right)$

$= 9.36 \times 10^{-4} \left(\frac{1 + 0.01 \times 7}{1 + 0.01 \times 4} \right) = 9.36 \times 10^{-4} \times 1.0288 =$

$= 9.63 \times 10^{-4} A$ when $V_o = 7$

*c$_1$) For $V_o = 2$ as $V_o < 3$, the right transistor is in Ohmic region

$I_o = \beta (2 (V_{GS} - V_{TO}) V_{DS} - V_{DS}^2)(1 + \lambda V_{DS})$

$= 10^{-4} \left(2(3)2 - 4 \right) (1 + 0.02) = 10^{-4} (8)(1.02)$

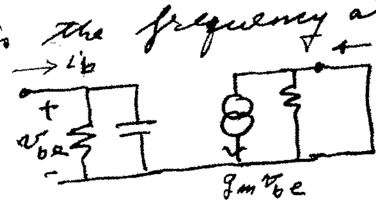
$= 8.16 \times 10^{-4} A$

#4. $g_m = I_E / V_T = - \frac{\alpha I_E}{V_T} = - \frac{\beta}{1+\beta} \cdot \frac{I_E}{V_T} = \frac{100}{101} \times \frac{50 \times 10^{-6}}{26 \times 10^{-3}} = 1.90 \times 10^{-3} \mho$

$g_\pi = \frac{g_m}{\beta} = \frac{1.90}{10^2} \times 10^{-3} = 1.90 \times 10^{-5} \mho$

$g_o = \frac{I_C}{V_A} = \frac{-\beta}{1+\beta} \frac{I_E}{V_A} = \frac{100}{101} \times \frac{50 \times 10^{-6}}{100} = 0.495 \times 10^{-6} \mho$

f_T is the frequency at which $|\beta(j\omega)| = 1$



$\Rightarrow \frac{i_c}{i_b} = \frac{g_m v_{be}}{(g_\pi + s C_\pi) v_{be}} = \frac{g_m}{s C_\pi + g_\pi}$

$= \beta(s)$

$|\beta(j\omega)| = \frac{g_m}{\sqrt{g_\pi^2 + C_\pi^2 \omega^2}} \Rightarrow 1 = \frac{g_m^2}{g_\pi^2 + C_\pi^2 \omega_T^2} \Rightarrow$

$\Rightarrow g_m^2 - g_\pi^2 = C_\pi^2 \omega_T^2$

$\Rightarrow \omega_T \approx g_m / C_\pi =$

$\Rightarrow f_T = \frac{1}{2\pi} \times \frac{g_m}{C_\pi} = \frac{1.90 \times 10^{-3}}{2\pi \times 20 \times 10^{-12}} = \frac{1.90 \times 10^{-3}}{1.256 \times 10^{-10}} = 1.513 \times 10^7$

$= 15.13 \times 10^6 Hz = 15.13$ Meg Hz

#5. Let $v_3 = v_{30}$, $v_2 = v_{20}$ then

1) $v_d = v_3 - v_2 = v - v_2$

as nodes 2 & 3 of op-amps draw no current

2) $v_2 = \frac{R_1}{R_1+R_2} v_0$

3) $i = sC(v_3 - v_0)$

4) = 2) → 1) $\quad v_d = v - \frac{R_1}{R_1+R_2} v_0$

5) = 3) + $v_3 = v$ ⇒ $i = sCv - sCv_0$

But

6) $v_0 = k v_d \equiv v_d = \frac{1}{k} v_0$

7) = 6) → 4) $\quad \frac{1}{k} v_0 = v - \frac{R_1}{R_1+R_2} v_0 \equiv \left(\frac{1}{k} + \frac{R_1}{R_1+R_2}\right) v_0 = v$

8) = 7) → 5

9) $\quad i = sCv - sC\left(\frac{v}{\frac{1}{k} + \frac{R_1}{R_1+R_2}}\right) v = sC\left\{1 - \frac{(R_1+R_2)}{\frac{1}{k}(R_1+R_2) + R_1}\right\} v$

a) 10) ∴ $\quad y = sC\left\{\frac{\frac{1}{k}(R_1+R_2) - R_2}{\frac{1}{k}(R_1+R_2) + R_1}\right\} = i/v$

For $k \to \infty$

b) $\quad y = -sC \times \frac{R_2}{R_1} = s\,C_{eq}$; $\quad C_{eq} = -\frac{R_2}{R_1} C$

The circuit gives a scaled negative capacitor
(so for stability would need $C > -C_{eq}$ in a parallel load)