Midterm Exam Add On Solution

a)

1) $v_3 = v$

KCL 2) $i = sC(v_3 - v_0) \Rightarrow i = sC(v - v_0)$

3) $v_0 = k\, v_d$

KVL 4) $v_d = v_3 - v_2 \Rightarrow v_d = v - v_2$

5) $v_2 = \dfrac{R_1}{R_1+R_2} v_0 \Rightarrow v_d = v - \dfrac{R_1}{R_1+R_2} v_0$

6) = 5)+3) $\Rightarrow v_0 = kv - k\dfrac{R_1}{R_1+R_2} v_0 \Rightarrow \left(1 + \dfrac{kR_1}{R_1+R_2}\right) v_0 = kv$

7) = 6)+2) $\Rightarrow i = sC\left(v - \dfrac{k}{1+\frac{kR_1}{R_1+R_2}} v\right) = sC\left(1 - \dfrac{(R_1+R_2)k}{(R_1+R_2)+kR_1}\right) v$

$$= sC\left(\frac{R_1+R_2-R_2 k}{R_1+R_2+kR_1}\right) v = sC\left(\frac{1/G_1 + 1/G_2 - k/G_2}{1/G_1 + 1/G_2 + k/G_1}\right) v$$

$$= sC\left(\frac{G_2+G_1-kG_1}{G_2+G_1+kG_2}\right) \cdot v$$

$$\Rightarrow \quad y(s) = sC\left(\frac{G_2+G_1-kG_1}{G_2+G_1+kG_2}\right)$$

b)

$$y(s) = sC\left(\frac{G_2+G_1 - \dfrac{k_\infty \sigma_1\sigma_2 G_1}{(s+\sigma_1)(s+\sigma_2)}}{G_2+G_1 + \dfrac{k_\infty \sigma_1\sigma_2 G_2}{(s+\sigma_1)(s+\sigma_2)}}\right)$$

$$= sC\left(\frac{(G_1+G_2)(s+\sigma_1)(s+\sigma_2) - k_\infty\sigma_1\sigma_2 G_1}{(G_1+G_2)(s+\sigma_1)(s+\sigma_2) + k_\infty\sigma_1\sigma_2 G_2}\right)$$

$$= sC\left(\frac{s^2 + (\sigma_1+\sigma_2)s + \sigma_1\sigma_2[1 - k_\infty G_1/(G_1+G_2)]}{s^2 + (\sigma_1+\sigma_2)s + \sigma_1\sigma_2[1 + k_\infty G_2/(G_1+G_2)]}\right)$$

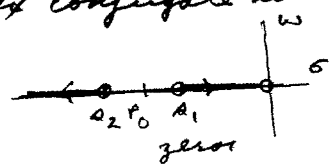
zeros: $s = 0$, $s_{1,2} = -\dfrac{(\sigma_1+\sigma_2)}{2} \pm \dfrac{1}{2}\sqrt{(\sigma_1+\sigma_2)^2 - 4\sigma_1\sigma_2[1 - k_\infty G_1/(G_1+G_2)]}$

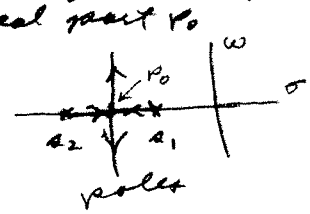
$$= -\frac{(\sigma_1+\sigma_2)}{2} \pm \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2 + 4\sigma_1\sigma_2 k_\infty G_1/(G_1+G_2)}$$

poles: $s_{3,4} = -\dfrac{(\sigma_1+\sigma_2)}{2} \pm \dfrac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2 - 4\sigma_1\sigma_2 k_\infty G_1/(G_1+G_2)}$

If $k_\infty = 0$ the poles cancel the non-zero zeros. As $k_\infty > 0$ increases the zeros remain real, moving away from $s_0 = -\dfrac{(\sigma_1+\sigma_2)}{2} \pm \dfrac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2}$ while the poles move toward $p_0 = -(\sigma_1+\sigma_2)/2$ and become complex conjugate with real part p_0

(for $k_\infty < 0$ the poles behave as the $k_\infty > 0$ nonzero zeros & vice versa)





c) If $k_\infty = 0$ $\quad y(s) = -sC \cdot G_1/G_2$ $\quad$ which is a capacitor of $C_{equivalent} = -(G_1/G_2)C$

When attached to a load $Y_L(s)$ instability can occur unless $Y_L(s)$ as $s \to \infty$ has $C_L > -C_{equivalent}$