s-plane $\quad P_1(D)\,y(t) = P_2(D)\,u(t) \quad D = d/dt$, u = input, y = output

$P(s)$ = polynomial of s. Then $u(t) = Ue^{st}$

$$y(t) = Ye^{st}$$

$$P_1(s)\,Ye^{st} = P_2(s)\,Ue^{st} \Rightarrow \frac{Y}{U} = \frac{P_2(s)}{P_1(s)}$$

Ex: $\quad \frac{d^2}{dt^2}y + a_1\frac{d}{dt}y + a_0 y = \frac{d}{dt}u + b_1 u \quad , -\infty < t < \infty$

$y = Ye^{st}$, $u = Ue^{st}$, $\quad \frac{d}{dt} = D$, $\frac{d}{dt}e^{st} = se^{st}$

$$\frac{d^2 e^{st}}{dt^2} = s^2 e^{st}$$

$$s^2 Y + a_1 s Y + a_0 Y = sU + b_1 U$$

$$\frac{Y}{U} = \frac{s + b_1}{s^2 + a_1 s + a_0} = \text{transfer function}$$

$$= \frac{P_2(s)}{P_1(s)} \Rightarrow P_2(s) = s + b_1, \text{ zero @ } s = -b_1, \infty$$

$P_1(s) = (s + k_1)(s + k_2)$, $s = -k_1, -k_2$ are the poles

$$= s^2 + a_1 s + a_0$$

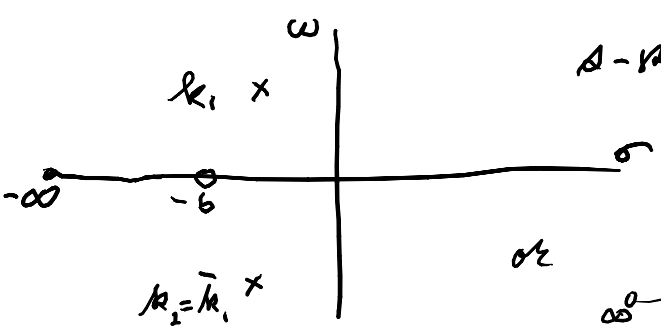
$$= s^2 + \frac{\omega_0}{Q}s + \omega_0^2$$

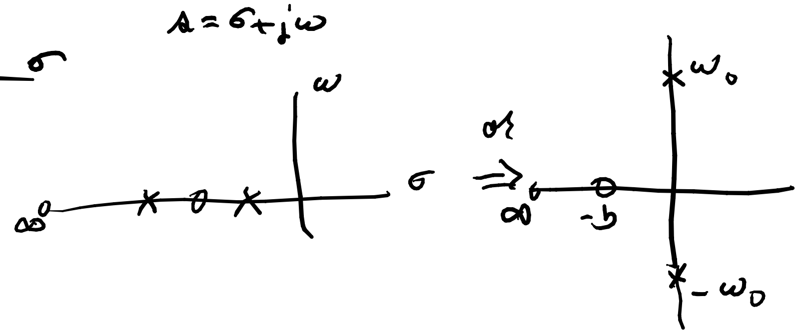
$$k_{1,2} = -\frac{a_1}{2} \pm \frac{1}{2}\sqrt{a_1^2 - 4a_0}$$

s-plane

$s = \sigma + j\omega$

$Q = \infty$





poles @ $s = \pm j\omega_0$

$$y(t) = Ae^{j\omega_0 t} + Be^{-j\omega_0 t}$$

$$= A(\cos\omega_0 t + j\sin\omega_0 t) + B(\cos(-\omega_0 t) + j\sin(-\omega_0 t))$$

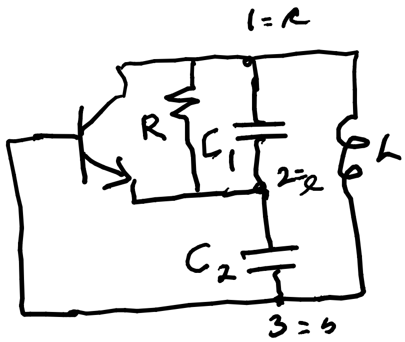
zeros give frequencies, $s = \sigma_1 + j\omega_1$, at which the system gives zero output poles give frequencies, $s=\sigma_2+j\omega_2$, at which the system responds in a natural way, due to initial conditions.

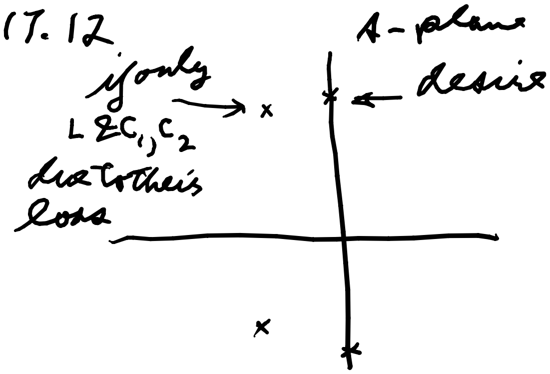
For stability wish poles in $\sigma<0$ or simple if on $\sigma=0$
$(s=\sigma+j\omega)$

polynomials with these zeros are called Hurwitz, or strictly Hurwitz if no poles on $\sigma=0$.

- - - -

Colpitts oscillator Fig. 17.12

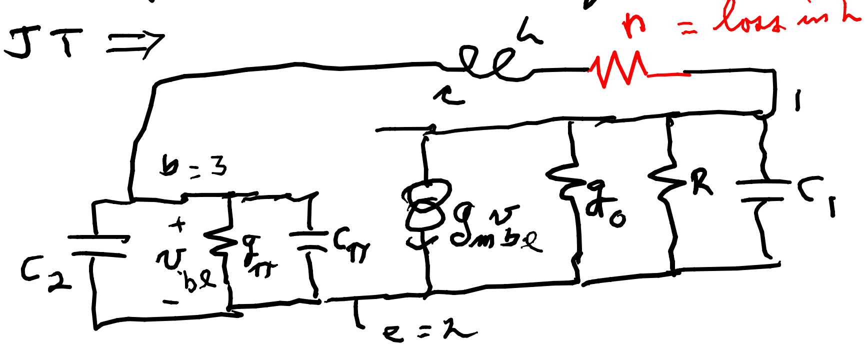




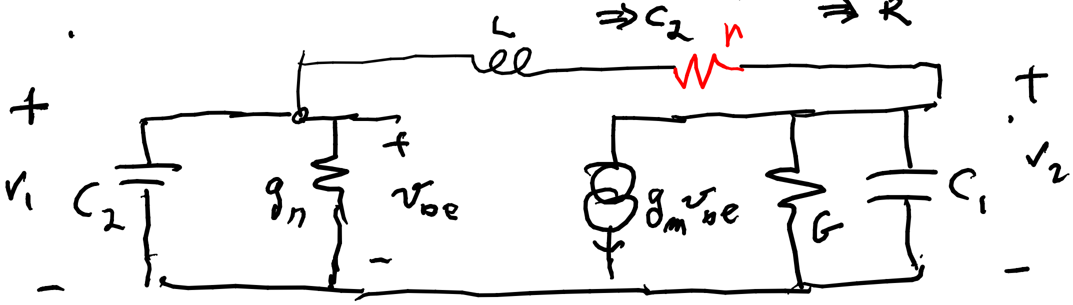
For small oscillations can use the π equivalent circuit of the BJT ⇒
SS eq. ckt.

$$g_m=\frac{I_C}{V_T},\quad g_o=\frac{I_C}{V_A}$$

$$g_\pi=\frac{g_m}{\beta}$$



can lump $C_2+C_\pi \Rightarrow C_2$, $R+r_o$ + loss of C_1 $\Rightarrow R$, loss of C_2 in g_π



$$Y_{11}=g_\pi+sC_2+\frac{1}{sL}+g$$

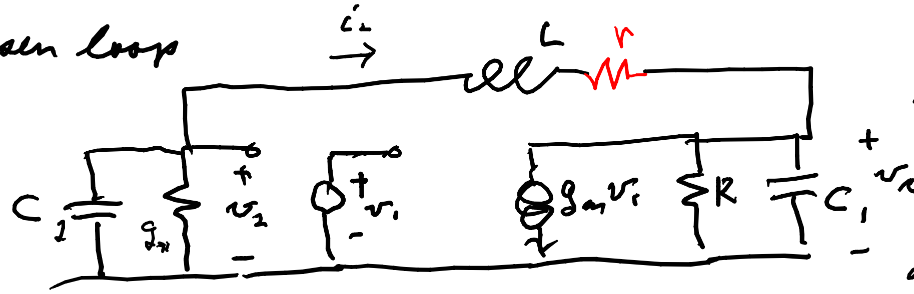
$$Y_{21}=g_m-\frac{1}{sL}-g$$

$$Y_{12}=-1/sL-g$$

$$Y_{22}=G+sC_1+\frac{1}{sL}+g$$

$g=1/r$

open loop



So we want $\frac{v_2}{v_1} = T(s)$

desire $v_2 = v_1$ for an oscillator.

$$(1a,b,c)\quad i_L = \frac{1}{(sL+r)}(v_2 - v_x) \overset{KCL}{=} g_m v_1 + G v_x + sC_1 v_x = g_m v_1 + (G+sC_1) v_x$$

$$(2)\qquad = -(g_\pi + sC_2) v_2 =$$

from (1)=(2) $\left[\frac{1}{sL+r} + (g_\pi + sC_2)\right] v_2 = \frac{1}{sL+r} v_x \Rightarrow$

$$(3)\qquad [1 + (sL+r)(g_\pi + sC_2)] v_2 = v_x$$

$$(3)\Rightarrow(1c)=(2)\qquad g_m v_1 = -\{(g_\pi + sC_2) + (G+sC_1)[1+(sL+r)(g_\pi+sC_2)]\} v_2$$

This gives the "return ratio" $T(s) = \frac{v_2}{v_1} = \frac{-g_m}{\{(g_\pi + sC_2) + (G+sC_1)[1+(sL+r)(g_\pi+sC_2)]\}}$

which has 3 zeros @ ∞, 3 finite poles

For the closed loop, $v_2 = v_1 \Rightarrow T(s) = 1$, which yields the "natural frequencies" as zeros of the degree 3 polynomial

$$P(s) = (g_\pi + sC_2) + (G+sC_1)[1+(sL+r)(g_\pi+sC_2)] + g_m$$

For sinusoidal oscillations, $s = j\omega_0$ so we want $P(j\omega_0) = 0$

$$P(j\omega_0) = (g_\pi + j\omega_0 C_2) + (G + j\omega_0 C_1) + (G+j\omega_0 C_1)(j\omega_0 L + r)(g_\pi + j\omega_0 C_2) + g_m$$

$$= (g_\pi + G) + j\omega_0(C_2 + C_1) + (j\omega_0 GL + Gr - \omega_0^2 LC_1 + j\omega_0 C_1 r)(g_\pi + j\omega_0 C_2) + g_m$$

$$= (g_\pi + G + g_m) + j\omega_0(C_2+C_1) + [j\omega_0 GLg_\pi - \omega_0^2 GLC_2 + Grg_\pi + j\omega_0 C_2 Gr$$
$$- \omega_0^2 LC_1 g_\pi - j\omega_0^3 LC_1C_2 + j\omega_0 C_1 r g_\pi - \omega_0^2 C_1C_2 r]$$

$$= (g_\pi + G + g_m - \omega_0^2 GLC_2 + Grg_\pi - \omega_0^2 LC_1 g_\pi - \omega_0^2 C_1C_2 r)$$
$$+ j\omega_0(C_2 + C_1 + GLg_\pi + C_2 Gr - \omega_0^2 LC_1C_2 + C_1 r g_\pi)$$

Real: $g_m = +\omega_0^2 LG + \{\omega_0^2 LC_1 g_\pi + \omega_0^2 C_1C_2 r\} - G - g_\pi - Grg_\pi$

Imag: $0 = -\omega_0^2 LC_1C_2 + C_2 + C_1 + GLg_\pi + C_2 G r + C_1 r g_\pi$

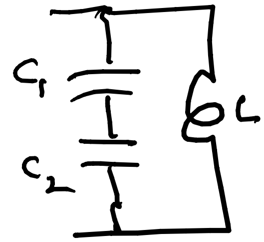
The imaginary part gives

$$\omega_0^2 = \frac{1}{L}\left(\frac{C_1+C_2}{C_1C_2}\right) + \underbrace{\left\{\frac{GLg_\pi + C_2 G r + C_1 g_\pi r}{LC_1C_2}\right\}}_{\text{correction due to losses over}}$$

end result ≈ $\omega_0 = \sqrt{\dfrac{1}{\dfrac{L C_1 C_2}{C_1 + C_2}}}$

Eq (17.16)

$$= \frac{1}{\sqrt{L C_{eq.}}}$$



and $\dfrac{g_m}{G} \approx C_2/C_1$ (eq. 17.21) gives gain, from real part