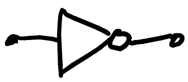
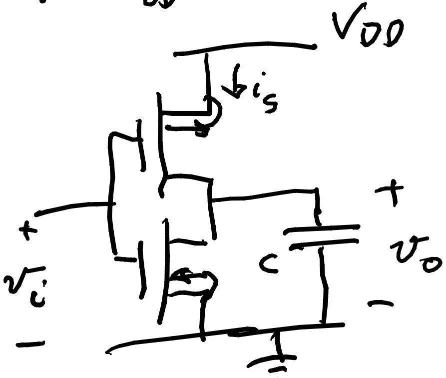
EE303H
10/11/11

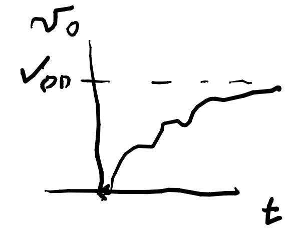
Note Title 10/10/2011

P. 1079 gives power dissipation

Eq. (13.35) $P_{dyn} = f C V_{DD}^2$







assume v_i switches from high (V_{DD}) to low (0) after C is totally discharged

Power from the battery is $V_{DD} i_S$

$$\text{Energy} = \int_0^t V_{DD}\, i_{S_p}(\tau)\, d\tau = V_{DD} \underbrace{\int_0^t i_{S_p}(\tau)\, d\tau}_{Q(t)} = V_{DD}\, C\, v_c(t)$$

Energy into a capacitor is

$$\int_0^t i_{cap} \cdot v_{cap}\, d\tau = \int_0^t C \frac{d v_{cap}}{d\tau} \cdot v_{cap}\, d\tau = \int_{v_c(0)}^{v_c(t)} C\, v_c\, dv_c$$

$$= \int_{v_c(0)}^{v(t)} C \frac{d v_c^2}{2} = \frac{C}{2} v_c^2 \Big|_{v_c(0)}^{v_c(t)} = \frac{C}{2} v_c^2(t); \quad Q = C v_c$$

if fully charge the capacitor $\Rightarrow v_c \to V_{DD}$

after fully switch from a high input to a low input then the Energy in the capacitor is $\frac{1}{2} C V_{DD}^2$

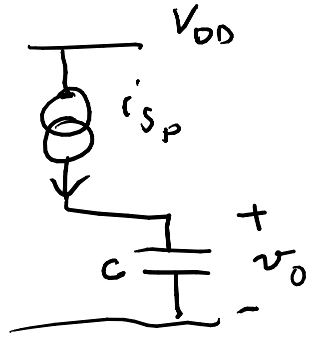
& the Energy out of the battery is $C V_{DD}^2$

i.e. the PMOS dissipates $\frac{1}{2} C V_{DD}^2$

if switch at frequency $f = 1/T$ (period $= T$) then double the energy in each cycle $\Rightarrow$ total energy lost in the CMOS inverter/cycle is CV_{DD}^2

for f cycles $\quad P_{diss} = f \cdot CV_{DD}^2 \qquad (17.35)$

Here



at $t = 0$

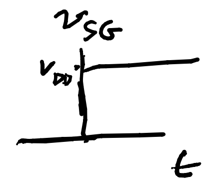
$v_{SG} = V_{DD}$

$v_O(0) = 0$

$= v_{Drain}$

here $v_{SD} = V_{DD} - v_O$

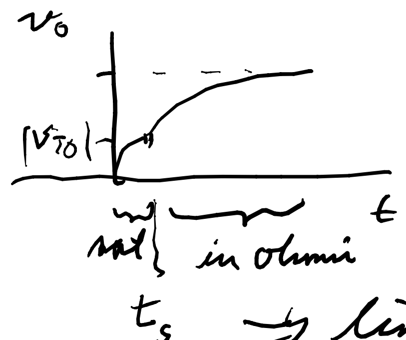
$v_{SG} = V_{DD}$



at $t = 0 \quad v_{SG} - |V_{TO}| = V_{DD} - |V_{TO}| \le V_{DD} = v_{SD}$

in saturation

remains in saturation until $v_O \Rightarrow$ gives $v_{SD} = V_{DD} - |V_{TO}|$



$v_O \Rightarrow i_{S_p} = C\dfrac{dv_O}{dt}$

for $v_O < |V_{TO}| \Rightarrow \dfrac{KP}{2}\dfrac{W}{L}(V_{DD} - |V_{TO}|)^2 = C\dfrac{dv_O}{dt}$

$\Rightarrow$ linear increase until t_S

after t_S then $i_S = \dfrac{KP}{2}\dfrac{W}{L}\left\{2(V_{DD} - |V_{TO}|)v_{SD} - v_{SD}^2\right\}$

$\beta = \dfrac{KP}{2}\dfrac{W}{L}, \quad v_{SD} = V_{DD} - v_O$

$$C\frac{dv_O}{dt} = \beta\left(2(V_{DD} - |V_{TO}|)(V_{DD} - v_O) - (V_{DD} - v_O)^2\right\}$$

a Riccati differential equation

$\dfrac{d(V_{DD} - v_O)}{dt} = -\dfrac{dv_O}{dt}$; $\quad x = V_{DD} - v_O \quad$ (varies between) $x(0) = V_{DD}$, $x(t_s) = 0$

$$-C\frac{dx}{dt} = \beta\left(2(V_{DD} - |V_{TO}|)x - x^2\right) = \beta\left(a x - x^2\right)$$

$$-\frac{c}{\beta}\cdot\frac{dx}{(ax-x^2)} = dt \Rightarrow \frac{dx}{x^2-ax} = \frac{\beta}{c}\,dt$$

$$\frac{\beta}{c}\int_0^{t_f} dt = \frac{\beta}{c}t_f = \int_{x(0)}^{x(t_f)} \frac{dx}{\underbrace{x^2-ax}_{x(x-a)}} = \int_{x(0)}^{x(t_f)}\left[\frac{k_1}{x}\,dx + \frac{k_2}{x-a}\,dx\right]$$

$$\frac{1}{x(x-a)} = \underbrace{\frac{k_1}{x} + \frac{k_2}{x-a}}_{\text{partial fraction expansion}} \Rightarrow \frac{\cancel{x}}{\cancel{x}(x-a)}\Bigg|_{x=0} = k_1 + \frac{k_2 x}{x-a}\Bigg|_{x=0} = \frac{1}{x-a}\Bigg|_{x=0} = \frac{1}{-a}$$

$$\frac{x-a}{x(x-a)}\Bigg|_{x=a} = \frac{1}{x}\Bigg|_{x=a} = \underbrace{\frac{k_1}{x}(x-a)}_{0} + k_2 \Rightarrow k_2 = 1/a$$

$$\frac{\beta}{c}\cdot t_f = \int_{x(0)}^{x(t_f)} \frac{-1/a}{x}\,dx + \int_{x(0)-a}^{x(t_f)-a} \frac{1/a}{x-a}\,d(x-a)$$