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EENE 610
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Set 4, Problem 5

5. a) $T(s) = \dfrac{2s+10}{s^2+3s+7}$ b) $T(s) = \dfrac{s^4+2s+10}{s^2+5s+6} = \dfrac{Y(s)}{U(s)}$

i) The impulse response: $u(t) = \delta(t)$ $\mathcal{L}(\delta(t)) = 1 = u(s)$

ii) The unit step response: $u(t) = u(t)$ $\mathcal{L}(u(t)) = 1/s = u(s)$

iii) The response due to initial conditions on the "finite" states when $u=0$: This will be the solution of the homogeneous equation: $\dot{x} = Ax$, $y = Cx$, $x = (sI-A)^{-1}x_0$

$y = C(sI-A)^{-1}x_0$

a i) $T(s) = \dfrac{2s+10}{s^2+3s+7}$ impulse response, $u(s) = 1$

$$Y(s) = \{C(sI-A)^{-1}B + D\}u(s)$$

$$A = \begin{bmatrix} 0 & 1 \\ -7 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [10, 2] \quad D = 0$$

$$(sI-A)^{-1} = \frac{1}{s^2+3s+7}\begin{bmatrix} s+3 & 1 \\ -7 & s \end{bmatrix}$$

$$Y(s) = \left\{[10, 2]\frac{1}{s^2+3s+7}\begin{bmatrix} s+3 & 1 \\ -7 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0\right\}(1) = T(s)$$

ROC $\sigma > -1.5$ $$Y(s) = \begin{bmatrix} \dfrac{10(s+3)-14}{s^2+3s+7}, & \dfrac{10+2s}{s^2+3s+7} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{2s+10}{s^2+3s+7} = \frac{2s+10}{(s+1.5)^2+(\sqrt{4.75})^2}$$

$$\mathcal{L}^{-1}\left[\frac{A_1 s + A_2}{(s+\alpha)^2+(\beta)^2}\right] = e^{-\alpha t}\left[A_1\cos\beta t + \frac{A_2-\alpha A_1}{\beta}\sin\beta t\right]u(t)$$

$$\boxed{y(t) = e^{-1.5t}\left[2\cos(\sqrt{4.75}\,t) + \frac{7}{\sqrt{4.75}}\sin(\sqrt{4.75}\,t)\right]u(t)}$$

a ii) Step response, $u = \frac{1}{s}$, $Y(s) = \frac{1}{s}(T(s))$

ROC $\sigma > 0$ $Y(s) = \frac{2s+10}{s(s^2+3s+7)}$ $y(t) = \mathcal{L}^{-1}(Y(s))$

$$\boxed{y(t) = \left[\frac{10}{7}\left(1 - e^{-1.5t}\right)\cos(\sqrt{4.75}\,t) - \left(\frac{4\sqrt{4.75}}{133}\right)e^{-1.5t}\sin(\sqrt{4.75}\,t)\right]u(t)}$$

b i) $T(s) = \frac{s^4+2s+10}{s^2+5s+6}$ impulse response, $u = 1$, $Y(s) = \cdot T(s)$

$$\begin{array}{r}
s^2 - 5s + 19 \\
(s^2+5s+6)\overline{\big)\, s^4 + 2s + 10} \\
-\;s^4 + 5s^3 + 6s^2 \\
\hline
0 - 5s^3 - 6s^2 + 2s + 10 \\
-\; -5s^3 - 25s^2 + 30s \\
\hline
0 \quad 19s^2 + 32s + 10 \\
-\; 19s^2 + 95s + 114 \\
\hline
0 \; -63s - 104
\end{array}$$

Re $\frac{-63s-104}{(s^2+5s+6)}$

ROC $\sigma > -2$ $Y(s) = s^2 - 5s + 19 + \frac{22}{(s+2)} - \frac{85}{(s+3)}$

$$\boxed{y(t) = \ddot{\delta}(t) - 5\dot{\delta}(t) + 19\delta(t) + \left[22e^{-2t} - 85e^{-3t}\right]u(t)}$$

b ii) Step response, $u = \frac{1}{s}$, $Y(s) = \frac{1}{s}(T(s))$

$Y(s) = \frac{s^4+2s+10}{s^3+5s^2+6s}$

$$\begin{array}{r}
s - 5 \\
(s^3+5s^2+6s)\overline{\big)\, s^4 + 2s + 10} \\
-\;s^4 + 5s^3 + 6s^2 \\
\hline
0 - 5s^3 - 6s^2 + 2s + 10 \\
-\; -5s^3 - 25s^2 - 30s + 10 \\
\hline
0 \quad 19s^2 + 32s + 10
\end{array}$$

Re $\frac{19s^2+32s+10}{(s^3+5s^2+6s)}$

ROC $\sigma > 0$ $Y(s) = (s-5) + \frac{1.667}{s} - \frac{11}{(s+2)} + \frac{28.333}{(s+3)}$

$$\boxed{y(t) = \dot{\delta}(t) - 5\delta(t) + \left[1.667 - 11e^{-2t} + 28.333e^{-3t}\right]u(t)}$$

iii) responses due to initial conditions when $u(s)=0$

a) $T(s) = \dfrac{2s+10}{s^2+3s+7}$

$$Y(s) = C(sI-A)^{-1}x_0 = \begin{bmatrix}10 & 2\end{bmatrix}\frac{1}{s^2+3s+7}\begin{bmatrix}s+3 & 1\\ -7 & s\end{bmatrix}\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}$$

$$Y(s) = \begin{bmatrix}\dfrac{10s+16}{s^2+3s+7}, & \dfrac{2s+10}{s^2+3s+7}\end{bmatrix}\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix} \quad \text{RoC } \sigma > -1.5$$

$$\boxed{\begin{aligned} y(t) = &\left[e^{-1.5t}\left(10\cos\sqrt{4.75}\,t + \frac{1}{\sqrt{4.75}}\sin\sqrt{4.75}\,t\right)\right]x_1(0)\,u(t) \;+ \\ &\left[e^{-1.5t}\left(2\cos\sqrt{4.75}\,t + \frac{7}{\sqrt{4.75}}\sin\sqrt{4.75}\,t\right)\right]x_2(0)\,u(t)\end{aligned}}$$

b) $T(s) = \dfrac{s^4+2s+10}{s^2+5s+6} = s^2-5s+19+\dfrac{-63s-104}{(s^2+5s+6)}$

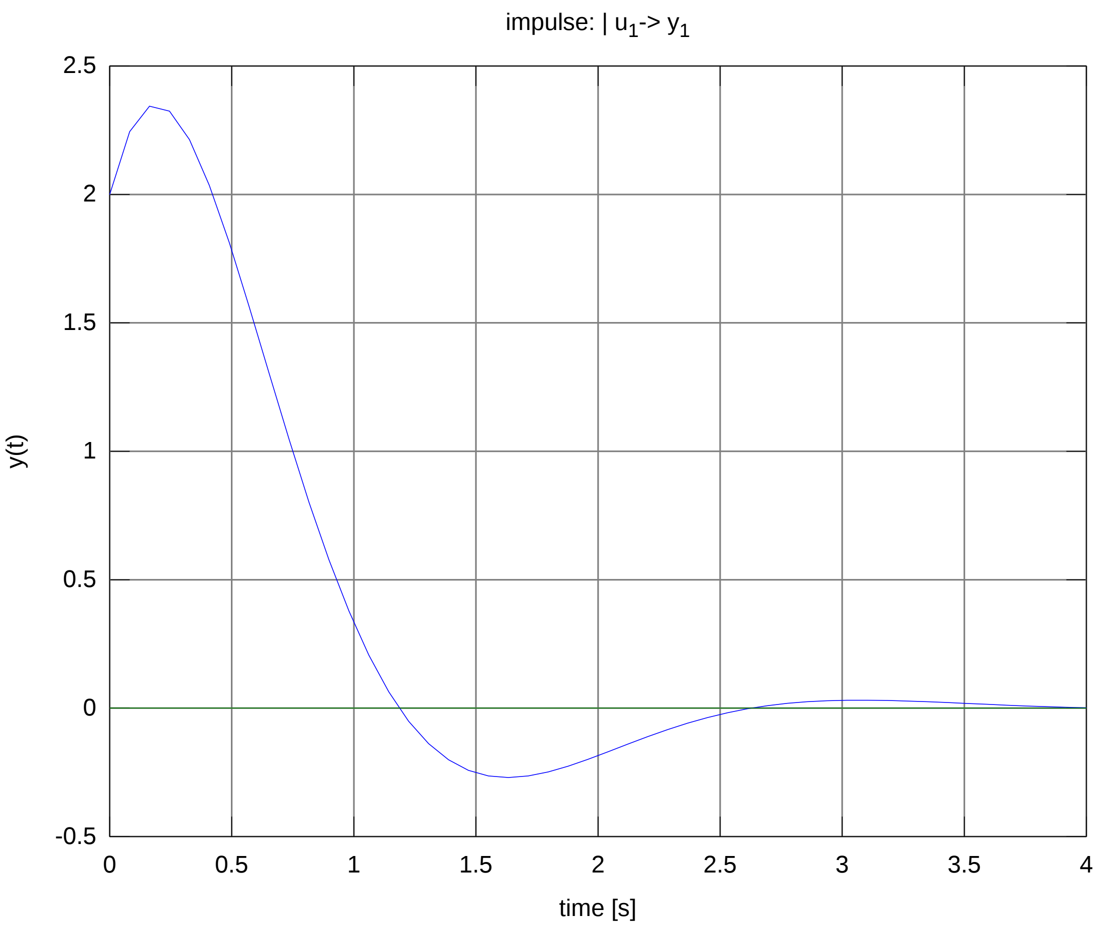
$$Y(s) = (s^2-5s+19) + \begin{bmatrix}-104 & -63\end{bmatrix}\frac{1}{(s^2+5s+6)}\begin{bmatrix}s+5 & 1\\ -6 & s\end{bmatrix}\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}$$

$$Y(s) = (s^2-5s+19) + \begin{bmatrix}\dfrac{-104s-142}{(s+2)(s+3)}, & \dfrac{-63s-104}{(s+2)(s+3)}\end{bmatrix}\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix} \quad \text{RoC } \sigma > -2$$

$$\boxed{\begin{aligned} y(t) = &\ddot{\delta}(t) - 5\dot{\delta}(t) + 19\delta(t) + \left[66e^{-2t} - 170e^{-3t}\right]x_1(0)\,u(t) \\ &+ \left[22e^{-2t} - 85e^{-3t}\right]x_2(0)\,u(t)\end{aligned}}$$

T(s) = 2s + 10/s^2+3s+7

```
octave:1> num=[0 2 10];
octave:2> den=[1 3 7];
octave:3> sys=tf(num,den);
octave:4> impulse(sys)
```



T(s) = 2s + 10/s^2 +3s +7

octave:5> step(sys)