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ENEE 610To do 4  
Problem 2

$$a) \quad y(s) = \frac{(s+4)(s+6)}{s+5}$$

① when  $s$  is a real, it is obvious that  $y(s)$  is also a real

$$② \quad \begin{array}{r} s+5 \overline{) s^2+10s+24} \\ \underline{s^2+5s} \phantom{+24} \\ 5s+24 \\ \underline{5s+25} \\ -1 \end{array}$$

$$\begin{aligned} s+5 - \frac{1}{s+5} &= (\sigma+j\omega)+5 - \frac{1}{(\sigma+j\omega)+5} \\ &= (\sigma+5) + j\omega - \frac{1}{(\sigma+5)+j\omega} \\ &= (\sigma+5) + j\omega - \frac{\sigma+5-j\omega}{(\sigma+5)^2+\omega^2} \end{aligned}$$

$$\begin{aligned} \operatorname{Re}\{y(s)\} &= \sigma+5 - \frac{\sigma+5}{(\sigma+5)^2+\omega^2} \\ &= \frac{(\sigma+5)((\sigma+5)^2+\omega^2) - (\sigma+5)}{(\sigma+5)^2+\omega^2} \\ &= \frac{(\sigma+5)(\sigma^2+10\sigma+24+\omega^2)}{(\sigma+5)^2+\omega^2} \geq 0, \sigma \geq 0 \end{aligned}$$

$$\boxed{\therefore y(s) \text{ is PR}}$$

$$b) \quad y(s) = \frac{s+5}{(s+4)(s+6)} \text{ is also a PR because}$$

$$\frac{1}{y(s)} \text{ is PR.}$$

$$c) \quad \text{1st Foster: } z(s) = \frac{1}{y(s)} = \frac{(s+4)(s+6)}{s+5}$$

$$\frac{s+5}{(s+4)(s+6)} = \frac{A}{s+4} + \frac{B}{s+6}$$

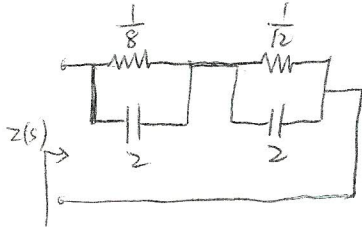
$$s+5 = A(s+6) + B(s+4)$$

Synthesize (a) and (b) by using 1st and 2nd Foster, 1st and 2nd Cauer and Richards function sections with  $k=1$ .

1st Foster

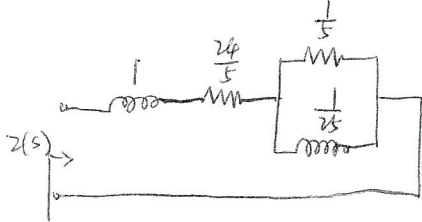
$$(a) \quad y(s) = \frac{(s+4)(s+6)}{s+5}$$

$$z(s) = \frac{s+5}{(s+4)(s+6)} = k_{\infty} + \frac{k_1}{s+4} + \frac{k_2}{s+6} \Rightarrow z(s) = \frac{1}{s+4} + \frac{1}{s+6} = \frac{1}{2s+8} + \frac{1}{2s+12}$$



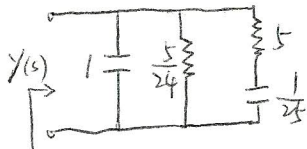
$$(b) \quad y(s) = \frac{s+5}{(s+4)(s+6)}$$

$$z(s) = \frac{(s+4)(s+6)}{s+5} = k_{\infty}s + \hat{k}_1 + \frac{\hat{k}_2s}{s+5} \Rightarrow k_{\infty}=1, \hat{k}_1 = \frac{24}{5}, \hat{k}_2 = \frac{1}{5} \Rightarrow z(s) = s + \frac{24}{5} + \frac{1}{5s}$$

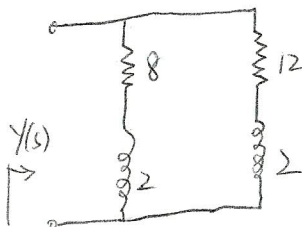


2nd Foster

$$(a) \quad y(s) = \frac{(s+4)(s+6)}{s+5} = k_{\infty}s + \hat{k}_1 + \frac{\hat{k}_2s}{s+5} = s + \frac{24}{5} + \frac{1}{5s}$$



$$(b) \quad y(s) = \frac{s+5}{(s+4)(s+6)} = \frac{1}{s+4} + \frac{1}{s+6} = \frac{1}{2s+8} + \frac{1}{2s+12}$$



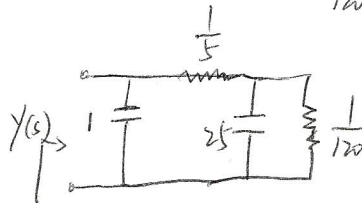
1st Camer

$$a) Y(s) = \frac{(s+4)(s+6)}{s+5} = \frac{s^2+10s+24}{s+5}$$

$$\begin{array}{r} s \\ s+5 \overline{) s^2+10s+24} \\ \underline{s+5s} \phantom{+24} \\ 5s+24 \phantom{+24} \\ \underline{5s+25} \\ \phantom{5s} -1 \phantom{+24} \end{array}$$

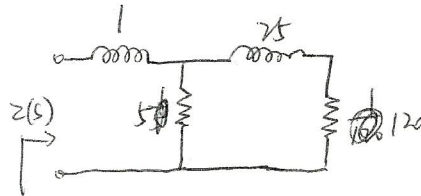
$$\frac{1}{s+5} = \frac{1}{5s+24} = \frac{25}{5s+24} = \frac{1}{\frac{1}{5}(5s+24)} = \frac{1}{5s+24}$$

$$\Rightarrow Y(s) = s + \frac{1}{\frac{1}{5} + \frac{1}{25s + \frac{1}{120}}}$$



$$b) Y(s) = \frac{s+5}{(s+4)(s+6)}$$

$$z(s) = \frac{(s+4)(s+6)}{s+5} = s + \frac{1}{\frac{1}{5} + \frac{1}{25s + \frac{1}{120}}}$$

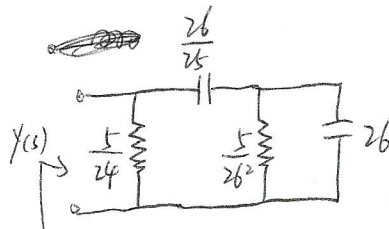


2nd Camer

$$a) Y(s) = \frac{(s+4)(s+6)}{s+5} = \frac{24+10s+s^2}{s+5}$$

$$\begin{array}{r} \frac{24}{5} \\ s+5 \overline{) 24+10s+s^2} \\ \underline{24+\frac{24}{5}s} \phantom{+s^2} \\ \phantom{24+} \frac{26}{5}s + s^2 \\ \underline{\phantom{24+} \frac{26}{5}s + \frac{26^2}{5}} \\ \phantom{24+} \frac{1}{26}s \phantom{+s^2} \\ \underline{\phantom{24+} \frac{26}{5}s} \\ \phantom{24+} \frac{1}{26}s \phantom{+s^2} \\ \underline{\phantom{24+} \frac{26}{5}s} \\ \phantom{24+} \frac{1}{26}s \phantom{+s^2} \end{array}$$

$$\Rightarrow Y(s) = \frac{1}{\frac{5}{24}} + \frac{1}{\frac{25}{26s} + \frac{1}{\frac{26^2}{5} + \frac{1}{26s}}}$$



$$b) Y(s) = \frac{s+5}{(s+4)(s+6)}$$

$$z(s) = \frac{(s+4)(s+6)}{s+5} = \frac{24+10s+s^2}{s+5} = \frac{1}{\frac{5}{24}} + \frac{1}{\frac{25}{26s} + \frac{1}{\frac{26^2}{5} + \frac{1}{26s}}}$$

