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PSet 4 #1. L-R properties

Using the partial fraction expansion for the first Foster reactance function derive the partial fraction expansion of the impedence z(s) for a passive inductor - resistor circuit. From that, give properties of the poles and zeros of z(s) and y(s) and the even part zeros of z(s). Relate the even part zeros of z(s) to those of y(s).

The first Foster function can be written as:

$$F_1(s) = K\frac{(s+\sigma_1)(s+\sigma_3)\cdots(s+\sigma_m)}{(s+\sigma_2)(s+\sigma_4)\cdots(s+\sigma_n)}$$

where $0 \leq \sigma_1 < \sigma_2 < \sigma_3 \cdots$ and $m = n \pm 1$.

The partial fraction expansion is therefore given as:

$$F_1(s) = K_1 + \frac{K_2 s}{s+\sigma_2} + \frac{K_4 s}{s+\sigma_4} + \cdots + \frac{K_n s}{s+\sigma_n}$$

This is in the form of a inductor - resistor network circuit since each term $\frac{Ks}{s+\sigma}$ can be considered an inductor-resistor pair in parallel: $z(s) = \frac{RLs}{R+Ls}$

$$z(s) = K\frac{(s+\sigma_1)(s+\sigma_3)\cdots(s+\sigma_m)}{(s+\sigma_2)(s+\sigma_4)\cdots(s+\sigma_n)}$$

Therefore, by looking at the partial fraction expansion of $z(s)$, we see that there are alternating poles and zeros at $s = -\sigma_1, -\sigma_3, \ldots, -\sigma_m$ (zeros) and $s = -\sigma_2, -\sigma_4, \ldots, -\sigma_n$ (poles).

$$y(s) = \frac{1}{z(s)} = \frac{1}{K}\frac{(s+\sigma_2)(s+\sigma_4)\cdots(s+\sigma_n)}{(s+\sigma_1)(s+\sigma_3)\cdots(s+\sigma_m)}$$

Looking at the partial fraction expansion of $y(s)$, we see that there are alternating poles and zeros at $s = -\sigma_1, -\sigma_3, \ldots, -\sigma_m$ (poles) and $s = -\sigma_2, -\sigma_4, \ldots, -\sigma_n$ (zeros).

To find the even part of the zeros of $z(s)$ we look at the admittance, $y(s)$ and use the formula $f_e(x) = \frac{1}{2}[f(x) + f(-x)]$:

$$y_e(s) = \frac{1}{2}[\frac{1}{K_\infty} + \frac{K_1 s}{s+\sigma_1} + \frac{K_3 s}{s+\sigma_3} + \cdots + \frac{K_m s}{s+\sigma_m} + \frac{1}{K_\infty} + \frac{K_1 s}{s-\sigma_1} + \frac{K_3 s}{s-\sigma_3} + \cdots + \frac{K_m s}{s-\sigma_m}]$$

This simplifies down to:

$$y_e(s) = \frac{1}{K_\infty} + \frac{K_1 s^2}{s^2-\sigma_1^2} + \frac{K_3 s^2}{s-\sigma_3^2} + \cdots + \frac{K_m s^2}{s-\sigma_m^2}$$

As you can see, the even part of $y(s)$ gives matching poles at $\pm\sigma_m$ which means that the even part of $z(s)$ has zeros at $\pm\sigma_m$. Similarly for $y(s)$, the even part will have zeros at $\pm\sigma_n$.