

November 23, 2010

#3

$$\bar{S}_1 = (\bar{Z} - \bar{R}_1)(\bar{Z} + \bar{R}_1)^{-1}$$

$$\bar{S}_2 = (\bar{Z} - \bar{R}_2)(\bar{Z} + \bar{R}_2)^{-1}$$

$$\bar{S}_1(\bar{Z} + \bar{R}_1) = (\bar{Z} - \bar{R}_1)$$

$$\bar{S}_1 \bar{Z} + \bar{S}_1 \bar{R}_1 = \bar{Z} - \bar{R}_1$$

$$\bar{S}_1 \bar{Z} - \bar{Z} = -\bar{S}_1 \bar{R}_1 + \bar{R}_1$$

$$\bar{Z} = \bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1} \text{ and } \bar{Z} = \bar{R}_2(\bar{S}_2 + \bar{I})(\bar{I} - \bar{S}_2)^{-1}$$

$$\Rightarrow \bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1} = \bar{R}_2(\bar{S}_2 + \bar{I})(\bar{I} - \bar{S}_2)^{-1}$$

$$\bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1}(\bar{I} - \bar{S}_2) = \bar{R}_2(\bar{S}_2 + \bar{I})$$

$$\bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1} - \bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1}\bar{S}_2 = \bar{R}_2\bar{S}_2 + \bar{R}_2$$

$$\bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1} - \bar{R}_2 = \bar{R}_2\bar{S}_2 + \bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1}\bar{S}_2$$

$$\bar{S}_2 = [\bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1} - \bar{R}_2] [\bar{R}_2 + \bar{R}_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1}]^{-1}$$

since $\bar{R}_1 = \begin{bmatrix} r_1 & 0 \\ 0 & r_1 \end{bmatrix}$ and $\bar{R}_2 = \begin{bmatrix} r_2 & 0 \\ 0 & r_2 \end{bmatrix}$

$$\bar{S}_2 = \left[r_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1} - r_2 \bar{I} \right] \left[r_2 \bar{I} + r_1(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1} \right]^{-1}$$

case when $r_1 = r_2$

$$\bar{S}_2 = \left[r_2(\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1} - (\bar{I} - \bar{S}_1)(\bar{I} - \bar{S}_1)^{-1} \right] \frac{1}{r_2} \left[(\bar{I} - \bar{S}_1)(\bar{I} - \bar{S}_1)^{-1} \right]$$

$$\bar{S}_2 = \left[(\bar{S}_1 + \bar{I}) - (\bar{I} - \bar{S}_1) \right] \cdot (\bar{I} - \bar{S}_1)^{-1} + (\bar{S}_1 + \bar{I})(\bar{I} - \bar{S}_1)^{-1}$$

$$\bar{S}_2 = 2 \cdot \bar{S}_1 (\bar{I} - \bar{S}_1)^{-1} \left[2 \cdot (\bar{I} - \bar{S}_1)^{-1} \right]^{-1}$$

$$\bar{S}_2 = 2 \cdot \bar{S}_1 (\bar{I} - \bar{S}_1)^{-1} \cdot \frac{1}{2} \cdot (\bar{I} - \bar{S}_1) = \bar{S}_1 \quad \checkmark$$