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ENEE 610
Problem Set 3.2

2. $y(s) = \dfrac{s^2 + bs + c}{s^2 + s + 1}$

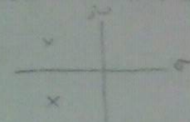
p. 339 - 340
Conditions for PR

(a) Show that this is pr for b & c both pos.

① No poles/zeros on right half of plane

$$\frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2} = -\frac{1}{2} \pm \frac{\sqrt{-3}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

roots on left half of plane



$$\frac{-b \pm \sqrt{b^2 - 4c}}{2} = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$$

✓ since b is pos, $-\frac{b}{2}$ is neg. so the real part of the "zero" will be on the left half of the plane

② poles on $j\omega$ axis are simple w/ pos. & real residues

⇒ no poles on $j\omega$ axis

③ real part of $z(j\omega)$ is pos. for $0 \le \omega \le \infty$

$$y(j\omega) = \frac{(j\omega)^2 + b(j\omega) + c}{(j\omega)^2 + j\omega + 1} = \frac{-\omega^2 + jb\omega + c}{-\omega^2 + j\omega + 1} = \frac{c - \omega^2 + jb\omega}{1 - \omega^2 + j\omega}$$

take complex conj. to reduce:

$$z(j\omega) = \frac{1}{Y(j\omega)} = \frac{(1-\omega^2) + j\omega}{(c-\omega^2) + jb\omega}\left(\frac{(c-\omega^2) - jb\omega}{(c-\omega^2) - jb\omega}\right) \Rightarrow \underbrace{\frac{(1-\omega^2)(c-\omega^2) + b\omega^2}{(c-\omega^2)^2 + b^2\omega^2}}_{Re\{z(j\omega)\}}$$

$$\mathrm{Re}\{Z(j\omega)\} = \frac{c-\omega^2-c\omega^2+\omega^4+b\omega^2}{c^2-2c\omega^2+\omega^4+b^2\omega^2}$$

$$= \frac{\omega^4+\omega^2(b-c-1)+c}{\omega^4+\omega^2(b^2-2c)+c^2}$$

PR for $0 \le \omega \le \infty$ given b & c are positive

(b) What if one or both are zero?

3 cases:

(i) b is zero
(ii) c is zero
(iii) both b & c are zero

case (i)

poles on $j\omega$ axis will not be simple w/ pos & real residues → not PR

case (ii)

conditions ① → ③ satisfied so PR

case (iii)

condition ② not satisfied so not PR

c) $\quad Ev\, Y(s) = \dfrac{Y(s)+Y(-s)}{2} = \dfrac{1}{2}\left(\dfrac{s^2+bs+c}{s^2+s+1} + \dfrac{s^2-bs+c}{s^2-s+1}\right)$

$$= \frac{1}{2}\left(\frac{(s^2+bs+c)(s^2-s+1)+(s^2-bs+c)(s^2+s+1)}{(s^2+s+1)(s^2-s+1)}\right)$$

$$= \frac{1}{2}\left(\frac{s^4 - \cancel{s^3} + s^2 + \cancel{bs^3} - bs^2 + \cancel{bs} + cs^2 - \cancel{cs} + c + s^4 + \cancel{s^3} + s^2 - \cancel{bs^3} - bs^2 - \cancel{bs} + cs^2 + \cancel{cs} + c}{(s^2+s+1)(s^2-s+1)}\right)$$

$$= \frac{1}{2} \times \frac{2s^4+2s^2-2bs^2+2cs^2+2c}{(s^2+s+1)(s^2-s+1)} = \frac{s^4+(c-b+1)s^2+c}{(s^2+s+1)(s^2-s+1)}$$

Let $s^4+(c-b+1)s^2+c=0, \Rightarrow s^2 = \dfrac{-(c-b+1)\pm\sqrt{(c-b+1)^2-4c}}{2}$

when $s^2 = \dfrac{-(c-b+1)-\sqrt{(c-b+1)^2-4c}}{2} = \dfrac{-\left[(c-b+1)+\sqrt{(c-b+1)^2-4c}\right]}{2}$

if $c-b+1<0$, $s^2>0$ since $c>0$, we get the real zeros $s=\pm\sqrt{\dfrac{-\left[(c-b+1)+\sqrt{(c-b+1)^2-4c}\right]}{2}}$

when $s^2 = \dfrac{-(c-b+1)+\sqrt{(c-b+1)^2-4c}}{2}$

also if $c-b+1<0$, $s^2>0$, we get the real zeros $s=\pm\sqrt{\dfrac{-(c-b+1)+\sqrt{(c-b+1)^2-4c}}{2}}$

Therefore, we finally determined when $c-b+1<0$, we have real zeros $s=\pm\sqrt{\dfrac{-(c-b+1)-\sqrt{(c-b+1)^2-4c}}{2}}$

and $s=\pm\sqrt{\dfrac{-(c-b+1)+\sqrt{(c-b+1)^2-4c}}{2}}$