Problem Set 3. Question 1.

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① @ If lossless, $I = S^{*T}S$, I = identity matrix

Means no energy lost. $*T$ = Conjugate Transpose.

For passive circuits, incapable of energy gain, $\therefore S^{*T}S \leq 1$

For active circuits, capable of energy gain, $\therefore S^{*T}S > 1$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$S^{*T} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$S^{*T}S = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Circuit is lossless & Passive ↓

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C^{*T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C^{*T}C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

also loseless & passive ↓

Bo Xiao

(1) (b)

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

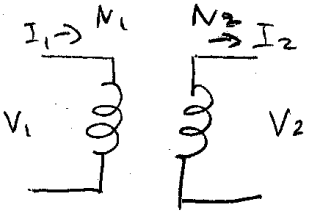
$H_{12} = H_{21} \Rightarrow$ reciprocal

$H_{11} = -H_{22} \Rightarrow$ Antimetric

Using ABCD Parameters:

for ideal transformers:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix}\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



Obtain ABCD Parameters from S-Parameters.

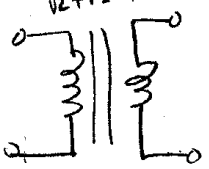
(www.rfcafe.com/references/electrical/s-h-y-z.htm)

$$A = \frac{(1+H_{11})(1-H_{22}) + H_{12}H_{21}}{2H_{21}} = \frac{(1+\frac{1}{\sqrt{2}})(1+\frac{1}{\sqrt{2}}) + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{2\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}+2}{\sqrt{2}} = \sqrt{2}+1$$

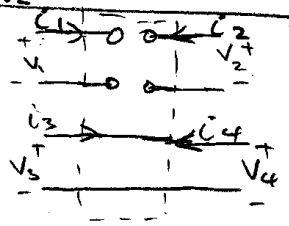
$$B = \frac{(1+H_{11})(1+H_{22}) - H_{12}H_{21}}{2H_{21}} = \frac{\frac{1}{2} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{2\frac{1}{\sqrt{2}}} = 0$$

$$C = \frac{(1-H_{11})(1-H_{22}) - H_{12}H_{21}}{2H_{21}} = \frac{\frac{1}{2} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{2\frac{1}{\sqrt{2}}} = 0$$

$$D = \frac{(1-H_{11})(1+H_{22}) + H_{12}H_{21}}{2H_{21}} = \frac{2-\sqrt{2}}{\frac{2}{\sqrt{2}}} = \sqrt{2}-1$$



$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



Similar to 3-port circulator, which is lossless