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ENEE 610
November 22, 2010

Pset 3 #1. S matrix for quantum computers

S matrix for quantum computers:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The above two (frequency independent) matricies are the key ones for quantum computing where H is for the Hadamard transformation and C is for the controlled not.

(a) Passive Circuits

Treating H and C as scattering matrices of a circuit, the circuit is passive if $I_n - S^{T*}S \geq 0$

For H:

$$I_2 - H^{T*}H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For C:

$$I_4 - C^{T*}C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

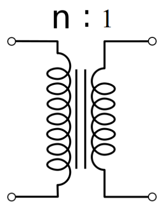
$\therefore$ not only are H and C passive, they are lossless as well.

(b) Passive Circuit Realization

For H, we notice that $H_{12} = H_{21}$, which means that it is reciprocal.
We also notice that $H_{11} = -H_{22}$, which means that it is antimetric.
This suggests a N:1 transformer with ABCD parameters of $\begin{bmatrix} N & 0 \\ 0 & 1/N \end{bmatrix}$.
Converting H from S parameters to ABCD parameters give:

$$A = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}} = \sqrt{2}+1$$
$$B = Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}} = 0$$
$$C = \frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}} = 0$$
$$D = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}} = \sqrt{2}-1$$

Therefore, H is equivalent to a $(\sqrt{2}+1) : 1$ transformer:



Looking at C, we see that it is similar to a 3-port circulator in that it is lossless and all energy from one input port is directed to one output port.
Since $S_{13}, S_{14}, S_{23}, S_{24}, S_{31}, S_{32}, S_{41}, S_{42}$ are all 0, we can reduce C into 2 separate matrices:

$$S^{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } S^{34} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Considering S^{12}, we see that it is the identity matrix so ports 1 and 2 are open circuits.
Looking at S^{34}, it is a pass-through so port 3 inputs will come out at port 4 and vice versa.