Problems to Consider Set 2

ENEE610
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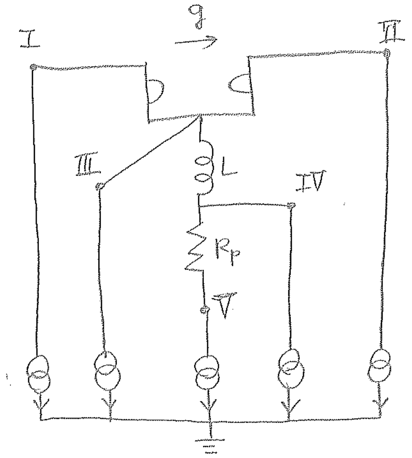
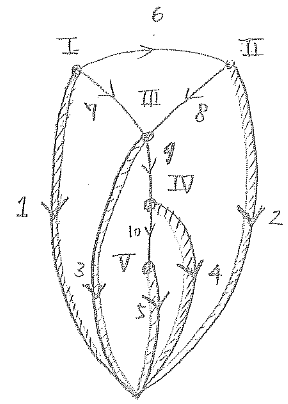
#3

(a)

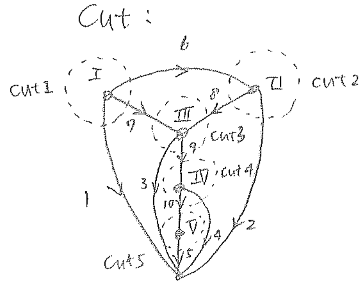


graph and tree $\Longrightarrow$



① Cut set matrix and tie set matrix:

Cut:



$$\vec{0} = \underbrace{\left[\begin{array}{ccccc:ccccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{array}\right]}\vec{i}$$

$\underline{\underline{C}}$: Cut set matrix

$\Rightarrow$ tie set matrix $\underline{\underline{T}} = \left[\begin{array}{ccccc:ccccc} -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$

② $\underline{\underline{A}}\,\vec{v} = \underline{\underline{B}}\,\vec{i}$:

$$\left[\begin{array}{ccccc:ccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_p} \end{array}\right]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \end{bmatrix} = \left[\begin{array}{ccccc:ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \\ i_{10} \end{bmatrix}$$

Use $\left[\,\underline{\underline{A}}\cdot\underline{\underline{C}}^T \;\vdots\; -\underline{\underline{B}}\cdot\underline{\underline{T}}^T\,\right]\begin{bmatrix} \vec{v}_t \\ \vec{i}_c \end{bmatrix} = -\underline{\underline{B}}\,\vec{J}$ $\Rightarrow$

$$\left[\begin{array}{ccccc:ccccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \hdashline 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & g & -g & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -g & 0 & g & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{sL} & \frac{-1}{sL} & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \frac{1}{R_p} & \frac{-1}{R_p} & 0 & 0 & 0 & 0 & -1 \end{array}\right]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \\ i_{10} \end{bmatrix} = \begin{bmatrix} -i_1 \\ -i_2 \\ -i_3 \\ -i_4 \\ -i_5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow i_6 = 0 \Rightarrow i_7 = -i_1, \quad i_8 = -i_2 \Rightarrow i_8 + i_2 + i_9 = -i_3, \quad -i_9 + i_{10} = -i_4, \quad i_{10} = i_5$

let $v_5 = 0 \Rightarrow \frac{1}{R_p} v_4 = i_{10}, \quad \frac{1}{sL}(v_3 - v_4) = -i_1 - i_2 - i_3, \quad g(-v_1 + v_3) = -i_2$

$g(v_2 - v_3) = -i_1$

③ the semistate equations:

(1) input current into port 1: $I_1 = -i_1 = g(v_2 - v_3) = g\left(v_2 - v_1 + \frac{i_2}{g}\right)$

(2) input current into port 2: $I_2 = -i_2$: $\frac{1}{sL}(v_3 - v_4) = -i_1 - i_2 - i_3$

$\Rightarrow \frac{1}{sL}\left[v_1 - \frac{i_2}{g} - R_p(-i_1 - i_2 - i_3 - i_4)\right] = -i_1 - i_2 - i_3$

choose $i_3 = i_4 = 0 \Rightarrow \frac{1}{sL}\left(v_1 - \frac{i_2}{g}\right) = \left(1 + \frac{R_p}{sL}\right)(-i_1 - i_2)$

$\because v_5 = 0 \quad \therefore$ The voltage at port 1: $V_1 = v_1$

The voltage at port 2: $V_2 = v_2$

$$\Rightarrow \begin{cases} V_1 = \frac{-1}{g} I_2 + sL(I_1 + I_2) + R_p(I_1 + I_2) \\ V_2 = +V_1 + \frac{1}{g} I_1 + \frac{1}{g} I_2 = \frac{1}{g} I_1 + sL(I_1 + I_2) + R_p(I_1 + I_2) \end{cases}$$

$\Rightarrow$ The semistate equations:

$$\boxed{\begin{cases} V_1 = \frac{-1}{g} I_2 + R_p(I_1 + I_2) + sL(I_1 + I_2) \\ V_2 = \frac{1}{g} I_1 + R_p(I_1 + I_2) + sL(I_1 + I_2) \end{cases}}$$

or in the matrix form: $\vec{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \vec{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = s \begin{bmatrix} L & L \\ L & L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} +R_p & +R_p - \frac{1}{g} \\ +R_p + \frac{1}{g} & +R_p \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(b) The two port impedance matrix:

$\because \vec{V} = \underline{\underline{Z}}\, \vec{I}$, the semistate equation: $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_p + sL & R_p + sL - \frac{1}{g} \\ R_p + sL + \frac{1}{g} & R_p + sL \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\therefore \boxed{\underline{\underline{Z}} = \begin{bmatrix} R_p + sL, & \frac{-1}{g} + R_p + sL \\ \frac{1}{g} + R_p + sL, & R_p + sL \end{bmatrix}}$$

Note the result is the same as # 1.