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Set 2

1a)

$$Y_{ind} = \left[\begin{array}{cc|cc|c} 0 & g & -g & 0 & 0 \\ -g & 0 & g & 0 & 0 \\ \hline g & -g & 1/sL & -1/sL & 0 \\ 0 & 0 & -1/sL & 1/sL + G_p & -G_p \\ \hline 0 & 0 & 0 & -G_p & G_p \end{array}\right]$$

Ground bottom node $\rightarrow$ set row, column 5 entries = 0.
Eliminate internal nodes by setting $I_{3,4} = i_3, i_4 = 0$

$$\begin{bmatrix} I_{1,2} \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_{1,2} \\ V_{3,4} \end{bmatrix}$$

$$I_{1,2} = Y_{11} V_{1,2} + Y_{12} V_{3,4}$$

$$0 = Y_{21} V_{1,2} + Y_{22} V_{3,4} \Rightarrow V_{3,4} = -Y_{22}^{-1} (Y_{21} V_{1,2})$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = I_{1,2} = (Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}) V_{1,2} = Y_{term} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{terminal} = Y_{2\text{-}port} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$= \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} - \begin{bmatrix} -g & 0 \\ g & 0 \end{bmatrix} \begin{bmatrix} \left(\frac{1}{G_p} + sL\right), & \left(\frac{1}{G_p}\right) \\ \left(1/G_p\right), & \left(1/G_p\right) \end{bmatrix} \begin{bmatrix} g & -g \\ 0 & 0 \end{bmatrix}$$

$$Y_{2\text{-}port} = \begin{bmatrix} g^2\left(\frac{1}{G_p + sL}\right), & g - g^2\left(\frac{1}{G_p + sL}\right) \\ -g - g^2\left(\frac{1}{G_p + sL}\right), & g^2\left(1/G_p + sL\right) \end{bmatrix}$$

invert Y to obtain $Z(s)$

$$\det Y_{2\text{-}port} = \left[g^2\left(\frac{1}{G_p + sL}\right)\right]^2 - (-g^2) - \left[g^2\left(\frac{1}{G_p + sL}\right)\right]^2$$

$$\det Y_{2\text{-}port} = g^2$$

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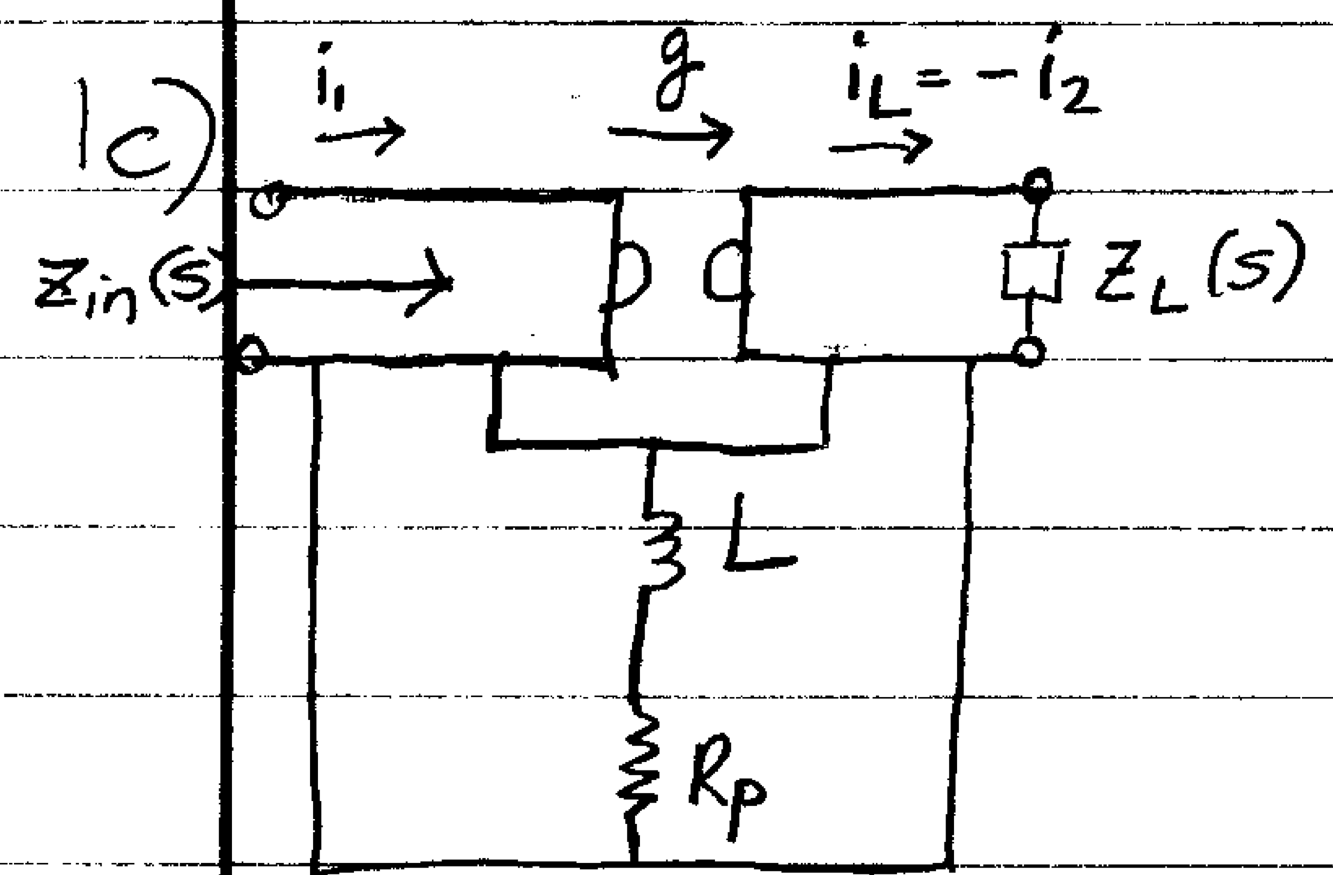
1b) $$Z(s) = \begin{bmatrix} \left(\frac{1}{G_p} + sL\right), & -\frac{1}{g} + \left(\frac{1}{G_p} + sL\right) \\ \frac{1}{g} + \left(\frac{1}{G_p} + sL\right), & \left(\frac{1}{G_p} + sL\right) \end{bmatrix} = Y^{-1}_{2\text{-port}}$$

$$Z(s) = \begin{bmatrix} R_p + sL, & -r + R_p + sL \\ r + R_p + sL, & R_p + sL \end{bmatrix} = Z_{gyr} \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$$

$$+ Z_{L+R_p} \begin{bmatrix} R_p + sL, & R_p + sL \\ R_p + sL, & R_p + sL \end{bmatrix}$$

$\Delta Z = r^2$

1c)



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad I_2 = -I_L$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = -Z_L I_2 = z_{21} I_1 + z_{22} I_2$$

$$I_2 = (z_L + z_{22})^{-1}(-z_{21} I_1)$$

$$V_1 = z_{11} I_1 + z_{12}(z_L + z_{22})^{-1}(-z_{21}) I_1 = z_{in}(s) I_1$$

$$z_{in}(s) = \frac{z_{11} z_L + \overbrace{(z_{11} z_{22} - z_{12} z_{21})}^{\Delta Z}}{z_L + z_{22}}$$

solve for $z_L(s)$

$$(z_{in}(s) - z_{11}) z_L(s) = \Delta Z - z_{in}(s) z_{22}$$

$$z_L(s) = \frac{\Delta Z - z_{in}(s) z_{22}}{z_{in}(s) - z_{11}} = \frac{r^2 - z_{in}(s)(R_p + sL)}{z_{in}(s) - (R_p + sL)}$$

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1d)

In $Z_L(s)$, let $R_p \to 0$ and identify $z_L(s)$ with $z_{in}(K)$ times a Richards' function, $R(s) = [K - s Z(s)]/[K Z(s) - s Z(K)]$

$$Z_L(s) = \frac{r^2 - Z_{in}(s) s L}{Z_{in}(s) - sL} = \frac{\left(\frac{r^2}{L}\right) - s Z_{in}(s)}{\left(\frac{1}{L}\right) Z_{in}(s) - s} = \frac{K z_{in}(K) - s Z_{in}(s)}{\frac{K}{Z_{in}(K)} Z_{in}(s) - s}$$

$$K z_{in}(K) = \left(\frac{r^2}{L}\right) = r^2\left(\frac{K}{Z_{in}(K)}\right) \to Z_{in}(K) = \frac{r^2}{Z_{in}(K)} \to r^2 = Z_{in}^2(K)$$

$$\frac{K}{Z_{in}}(K) = \left(\frac{1}{L}\right)$$