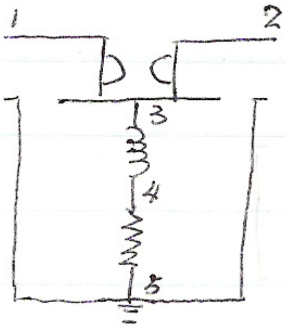
ENEE610 Minshu Zhao

Oct.25 2010

Problem set 2 Question 1

a)



indefinate admittance

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \\ \end{array}\begin{bmatrix} 0 & g & -g & 0 & 0 \\ -g & 0 & g & 0 & 0 \\ g & -g & \frac{1}{sL} & -\frac{1}{sL} & 0 \\ 0 & 0 & -\frac{1}{sL} & \frac{1}{sL}+\frac{1}{R_p} & -\frac{1}{R_p} \\ 0 & 0 & 0 & -\frac{1}{R_p} & \frac{1}{R_p} \end{bmatrix}$$
$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}$$

nodal admittance

$$\begin{bmatrix} 0 & g & -g & 0 \\ -g & 0 & g & 0 \\ g & -g & \frac{1}{sL} & -\frac{1}{sL} \\ 0 & 0 & -\frac{1}{sL} & \frac{1}{sL}+\frac{1}{R_p} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

2port admittance $Y(s)$

let $i_3 = i_4 = 0$ as intenal nodes

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \left(Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}\right)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \left[\begin{pmatrix} 0 & g \\ -g & 0 \end{pmatrix} - \begin{pmatrix} -g & 0 \\ g & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{sL} & \frac{1}{sL}+\frac{1}{R_p} \end{pmatrix}^{-1}\begin{pmatrix} g & -g \\ 0 & 0 \end{pmatrix}\right]\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \left[\begin{pmatrix} 0 & g \\ -g & 0 \end{pmatrix} - \begin{pmatrix} -g & 0 \\ g & 0 \end{pmatrix}\begin{pmatrix} sL+R_p & R_p \\ R_p & R_p \end{pmatrix}\begin{pmatrix} g & -g \\ 0 & 0 \end{pmatrix}\right]\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \left[\begin{pmatrix} 0 & g \\ -g & 0 \end{pmatrix} - \begin{pmatrix} -g^2sL-g^2R_p & g^2sL+g^2R_p \\ g^2sL+g^2R_p & -g^2sL-g^2R_p \end{pmatrix}\right]\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \begin{bmatrix} g^2sL+g^2R_p & g-g^2sL-g^2R_p \\ -g-g^2sL-g^2R_p & g^2sL+g^2R_p \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

b) $$Z(s) = Y^{-1}(s) = \begin{bmatrix} sL + R_p & -\frac{1}{g} + sL + R_p \\ \frac{1}{g} + sL + R_p & sL + R_p \end{bmatrix}$$

check: $V_1 = -\frac{1}{g} i_2 + (sL + R_p)(i_2 + i_1)$

$V_2 = \frac{1}{g} i_1 + (sL + R_p)(i_1 + i_2)$

c) $$Y_L(s) = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{z_L(s)} \end{bmatrix}$$

$$\Rightarrow Y_{in} = \begin{bmatrix} g^2 sL + g^2 R_p & g - g^2 sL - g^2 R_p \\ -g - g^2 sL - g^2 R_p & g^2 sL + g^2 R_p + \frac{1}{z_L(s)} \end{bmatrix}$$

$$z_{in}(s) = Z_{in}(s)_{11} = \frac{g^2 (sL + R_p) z_L(s) + 1}{g^2 z_L(s) + (sL + R_p) g^2}$$

$$\Rightarrow z_L(s) = \frac{g^2 z_{in}(s)(sL + R_p) - 1}{(sL + R_p) g^2 - g^2 z_{in}(s)}$$

d) let $R_p = 0$

$$z_L(s) = \frac{g^2 z_{in} sL - 1}{sL g^2 - g^2 z_{in}(s)}$$

from Richards function: $R(s) = \dfrac{k - s Z(s)}{k Z(s) - s Z(k)}$

$$z_{in}(k) R(s) = \frac{z_{in}(k) k - s z_{in}(k) z_{in}(s)}{k z_{in}(s) - s z(k)} = z_L(s)$$

$z_{in}(k)\, k = 1$

$\Rightarrow z_{in}(s) z_{in}(k) = L g^2 z(s)$

$g^2 z_{in}(s) = k z_{in}(s)$

$s z_{in}(k) = s L g^2$

$\Rightarrow$

$g^2 = k$

$L g^2 = Z_{in}$

$L g^4 = 1$