HW Set 1
Problem 5:

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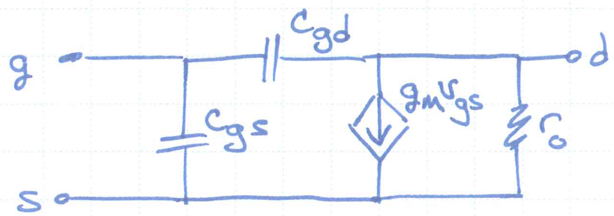
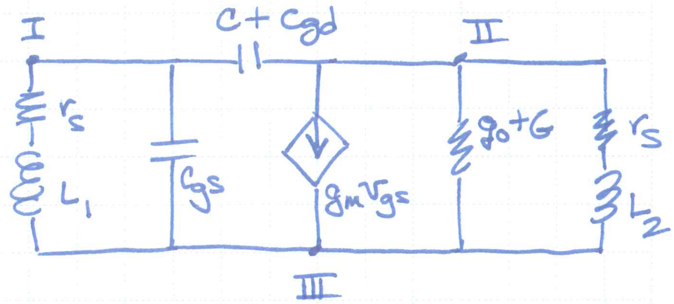
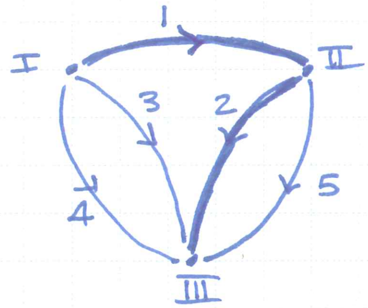
small signal model for MOSFET



small signal model for Hartley oscillator



$$\bar{Q}_f = \left[\begin{array}{cc:ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right] \quad \text{cutset matrix}$$



tree: 1, 2

$$\bar{B}_f = \left[\begin{array}{cc:ccc} -1 & -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{array}\right] \quad \text{tieset matrix}$$

Voltage-current relationships:

$$\begin{bmatrix} s(C+C_{gd}) & 0 & 0 & 0 & 0 \\ 0 & (g_o+G) & g_m & 0 & 0 \\ 0 & 0 & sC_{gs} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & (sL_1+r_s) & 0 \\ 0 & 0 & 0 & 0 & (sL_2+r_s) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

Since there are no independent voltage or current sources, $\bar{v}_s = \bar{0}$ and $\bar{i}_s = \bar{0}$

$$\Rightarrow \bar{v} = \bar{v}_b \text{ and } \bar{i} = \bar{i}_b$$

$$\left[\bar{A}(s)\cdot \bar{Q}_f^T \;\vdots\; -\bar{B}(s)\cdot \bar{B}_f^T \right]\cdot \begin{bmatrix} \bar{v}_t \\ \cdots \\ \bar{i}_c \end{bmatrix} = \bar{0}_{5\times 5}$$

$$\left[\begin{array}{cc:ccc} s(C+C_{gd}) & 0 & 1 & 1 & 0 \\ g_m & (g_m+G_o) & 1 & 1 & 1 \\ sC_{gs} & sC_{gs} & -1 & 0 & 0 \\ 1 & 1 & 0 & -(sL_1+r_s) & 0 \\ 0 & 1 & 0 & 0 & -(sL_2+r_s) \end{array}\right] \bar{x} = \bar{0}_{5\times 5}$$

branch voltages & currents

$$\text{where } \bar{x} = \begin{bmatrix} \bar{v}_t \\ \cdots \\ \bar{i}_c \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

$$s\begin{bmatrix} (C+C_{gd}) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ C_{gs} & C_{gs} & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_1 & 0 \\ 0 & 0 & 0 & 0 & -L_2 \end{bmatrix} \bar{x} = \begin{bmatrix} 0 & 0 & -1 & -1 & 0 \\ -g_m & -(g_m+G_o) & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & r_s & 0 \\ 0 & -1 & 0 & 0 & r_s \end{bmatrix} \bar{x}$$

Row 2: $0 = g_m v_1 + G_m v_2 + i_3 + i_4 + i_5$

Rows 1 & 2: $s(C+C_{gd})v_1 = g_m v_1 + G_m v_2 + i_5$

Rows 1, 2, & 5: $[s(C+C_{gd}) - g_m]v_1 = G_m(sL_2+r_s)i_5 + i_5$

$s(C+C_{gd})v_1 - sG_m L_2 i_5 = g_m v_1 + (G_m r_s + 1) i_5$ ✓

Rows 4 & 5: $sL_1 i_4 - sL_2 i_5 = v_1 - r_s i_4 + r_s i_5$ ✓

Rows 3 & 4: $s^2 L_1 C_{gs} i_4 = i_3 - s r_s C_{gs} i_4$

Rows 1, 3, & 4: $s^2 L_1 C_{gs} i_4 + s(C+C_{gd})v_1 + s r_s C_{gs} i_4 = -i_4$ ✓

$$s^2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & L_1 C_{gs} & 0 \end{bmatrix}\underline{x} + s\begin{bmatrix} C+C_{gd} & 0 & -G_m L_2 \\ 0 & L_1 & -L_2 \\ C+C_{gd} & r_s C_{gs} & 0 \end{bmatrix}\underline{x} = \begin{bmatrix} g_m & 0 & G_m r_s + 1 \\ 1 & -r_s & r_s \\ 0 & -1 & 0 \end{bmatrix}\underline{x}$$

where $\underline{x} = [v_1 \;\; i_4 \;\; i_5]^T$

The previous matrix equation is in the following form:

$$s^2 \cdot D \cdot \underline{x} + s \cdot E \cdot \underline{x} = A \cdot \underline{x}$$

The oscillation conditions can be determined by solving the following equation:

$$s^2 \cdot \mathbf{D} + s \cdot \mathbf{E} - \mathbf{A} = \mathbf{0}$$

At this point MATLAB (or a similar software package) may be used to assist in solving the above equation. Taking the determinant of both sides of the equation gives the following result:

```
0 = (C*Cgs*L1*L2 + Cgd*Cgs*L1*L2)*s^4 + (C*Gm*L1*L2 + Cgd*Gm*L1*L2 + Cgs*Gm*L1*L2
- Cgs*L1*L2*gm + C*Cgs*L1*r + C*Cgs*L2*r + Cgd*Cgs*L1*r + Cgd*Cgs*L2*r)*s^3 +
(C*L1 + C*L2 + Cgd*L1 + Cgs*L1 + Cgd*L2 + C*Cgs*r^2 + Cgd*Cgs*r^2 + C*Gm*L1*r +
C*Gm*L2*r + Cgd*Gm*L1*r + Cgs*Gm*L1*r + Cgd*Gm*L2*r + Cgs*Gm*L2*r - Cgs*L1*gm*r -
Cgs*L2*gm*r)*s^2 + (2*C*r - L2*gm + 2*Cgd*r + Cgs*r + Gm*L2 + C*Gm*r^2 +
Cgd*Gm*r^2 + Cgs*Gm*r^2 - Cgs*gm*r^2)*s + Gm*r - gm*r + 1
```

Note: "r" here corresponds to the series resistance of the inductors and was previously denoted by "r_s".

Substitute s=j*omega

```
0 = (C*Cgs*L1*L2 + Cgd*Cgs*L1*L2)*omega^4 - j*(C*Gm*L1*L2 + Cgd*Gm*L1*L2 +
Cgs*Gm*L1*L2 - Cgs*L1*L2*gm + C*Cgs*L1*r + C*Cgs*L2*r + Cgd*Cgs*L1*r +
Cgd*Cgs*L2*r)*omega^3 - (C*L1 + C*L2 + Cgd*L1 + Cgs*L1 + Cgd*L2 + C*Cgs*r^2 +
Cgd*Cgs*r^2 + C*Gm*L1*r + C*Gm*L2*r + Cgd*Gm*L1*r + Cgs*Gm*L1*r + Cgd*Gm*L2*r +
Cgs*Gm*L2*r - Cgs*L1*gm*r - Cgs*L2*gm*r)*omega^2 + j*(2*C*r - L2*gm + 2*Cgd*r +
Cgs*r + Gm*L2 + C*Gm*r^2 + Cgd*Gm*r^2 + Cgs*Gm*r^2 - Cgs*gm*r^2)*omega + Gm*r -
gm*r + 1
```

Separate into two equations (real and imaginary parts)

```
0 = (C*Cgs*L1*L2 + Cgd*Cgs*L1*L2)*omega^4 - (C*L1 + C*L2 + Cgd*L1 + Cgs*L1 +
Cgd*L2 + C*Cgs*r^2 + Cgd*Cgs*r^2 + C*Gm*L1*r + C*Gm*L2*r + Cgd*Gm*L1*r +
Cgs*Gm*L1*r + Cgd*Gm*L2*r + Cgs*Gm*L2*r - Cgs*L1*gm*r - Cgs*L2*gm*r)*omega^2 +
Gm*r - gm*r + 1

0 = -(C*Gm*L1*L2 + Cgd*Gm*L1*L2 + Cgs*Gm*L1*L2 - Cgs*L1*L2*gm + C*Cgs*L1*r +
C*Cgs*L2*r + Cgd*Cgs*L1*r + Cgd*Cgs*L2*r)*omega^3 + (2*C*r - L2*gm + 2*Cgd*r +
Cgs*r + Gm*L2 + C*Gm*r^2 + Cgd*Gm*r^2 + Cgs*Gm*r^2 - Cgs*gm*r^2)*omega
```

Solve the equation from real part for omega, the oscillation frequency:

```
omega = ((C*L1 - (C^2*Cgs^2*r^4 + 2*C^2*Cgs*Go*L1*r^3 + 2*C^2*Cgs*Go*L2*r^3 +
2*C^2*Cgs*L1*gm*r^3 + 2*C^2*Cgs*L1*r^2 + 2*C^2*Cgs*L2*gm*r^3 + 2*C^2*Cgs*L2*r^2 +
C^2*Go^2*L1^2*r^2 + 2*C^2*Go^2*L1*L2*r^2 + C^2*Go^2*L2^2*r^2 +
2*C^2*Go*L1^2*gm*r^2 + 2*C^2*Go*L1^2*r + 4*C^2*Go*L1*L2*gm*r^2 + 4*C^2*Go*L1*L2*r
+ 2*C^2*Go*L2^2*gm*r^2 + 2*C^2*Go*L2^2*r + C^2*L1^2*gm^2*r^2 + 2*C^2*L1^2*gm*r +
C^2*L1^2 + 2*C^2*L1*L2*gm^2*r^2 + 4*C^2*L1*L2*gm*r + 2*C^2*L1*L2 +
C^2*L2^2*gm^2*r^2 + 2*C^2*L2^2*gm*r + C^2*L2^2 + 2*C*Cgd*Cgs^2*r^4 +
4*C*Cgd*Cgs*Go*L1*r^3 + 4*C*Cgd*Cgs*Go*L2*r^3 + 4*C*Cgd*Cgs*L1*gm*r^3 +
4*C*Cgd*Cgs*L1*r^2 + 4*C*Cgd*Cgs*L2*gm*r^3 + 4*C*Cgd*Cgs*L2*r^2 +
2*C*Cgd*Go^2*L1^2*r^2 + 4*C*Cgd*Go^2*L1*L2*r^2 + 2*C*Cgd*Go^2*L2^2*r^2 +
4*C*Cgd*Go*L1^2*gm*r^2 + 4*C*Cgd*Go*L1^2*r + 8*C*Cgd*Go*L1*L2*gm*r^2 +
8*C*Cgd*Go*L1*L2*r + 4*C*Cgd*Go*L2^2*gm*r^2 + 4*C*Cgd*Go*L2^2*r +
2*C*Cgd*L1^2*gm^2*r^2 + 4*C*Cgd*L1^2*gm*r + 2*C*Cgd*L1^2 + 4*C*Cgd*L1*L2*gm^2*r^2
+ 8*C*Cgd*L1*L2*gm*r + 4*C*Cgd*L1*L2 + 2*C*Cgd*L2^2*gm^2*r^2 + 4*C*Cgd*L2^2*gm*r +
2*C*Cgd*L2^2 + 2*C*Cgs^2*Go*L1*r^3 + 2*C*Cgs^2*Go*L2*r^3 + 2*C*Cgs^2*L1*r^2 +
2*C*Cgs*Go^2*L1^2*r^2 + 4*C*Cgs*Go^2*L1*L2*r^2 + 2*C*Cgs*Go^2*L2^2*r^2 +
2*C*Cgs*Go*L1^2*gm*r^2 + 4*C*Cgs*Go*L1^2*r + 4*C*Cgs*Go*L1*L2*gm*r^2 +
2*C*Cgs*Go*L1*L2*r + 2*C*Cgs*Go*L2^2*gm*r^2 + 2*C*Cgs*Go*L2^2*r +
2*C*Cgs*L1^2*gm*r + 2*C*Cgs*L1^2 + 2*C*Cgs*L1*L2*gm*r - 2*C*Cgs*L1*L2 +
Cgd^2*Cgs^2*r^4 + 2*Cgd^2*Cgs*Go*L1*r^3 + 2*Cgd^2*Cgs*Go*L2*r^3 +
2*Cgd^2*Cgs*L1*gm*r^3 + 2*Cgd^2*Cgs*L1*r^2 + 2*Cgd^2*Cgs*L2*gm*r^3 +
2*Cgd^2*Cgs*L2*r^2 + Cgd^2*Go^2*L1^2*r^2 + 2*Cgd^2*Go^2*L1*L2*r^2 +
Cgd^2*Go^2*L2^2*r^2 + 2*Cgd^2*Go*L1^2*gm*r^2 + 2*Cgd^2*Go*L1^2*r +
4*Cgd^2*Go*L1*L2*gm*r^2 + 4*Cgd^2*Go*L1*L2*r + 2*Cgd^2*Go*L2^2*gm*r^2 +
2*Cgd^2*Go*L2^2*r + Cgd^2*L1^2*gm^2*r^2 + 2*Cgd^2*L1^2*gm*r + Cgd^2*L1^2 +
2*Cgd^2*L1*L2*gm^2*r^2 + 4*Cgd^2*L1*L2*gm*r + 2*Cgd^2*L1*L2 + Cgd^2*L2^2*gm^2*r^2
+ 2*Cgd^2*L2^2*gm*r + Cgd^2*L2^2 + 2*Cgd*Cgs^2*Go*L1*r^3 + 2*Cgd*Cgs^2*Go*L2*r^3 +
2*Cgd*Cgs^2*L1*r^2 + 2*Cgd*Cgs*Go^2*L1^2*r^2 + 4*Cgd*Cgs*Go^2*L1*L2*r^2 +
2*Cgd*Cgs*Go^2*L2^2*r^2 + 2*Cgd*Cgs*Go*L1^2*gm*r^2 + 4*Cgd*Cgs*Go*L1^2*r +
4*Cgd*Cgs*Go*L1*L2*gm*r^2 + 2*Cgd*Cgs*Go*L1*L2*r + 2*Cgd*Cgs*Go*L2^2*gm*r^2 +
2*Cgd*Cgs*Go*L2^2*r + 2*Cgd*Cgs*L1^2*gm*r + 2*Cgd*Cgs*L1^2 + 2*Cgd*Cgs*L1*L2*gm*r
- 2*Cgd*Cgs*L1*L2 + Cgs^2*Go^2*L1^2*r^2 + 2*Cgs^2*Go^2*L1*L2*r^2 +
Cgs^2*Go^2*L2^2*r^2 + 2*Cgs^2*Go*L1^2*r + 2*Cgs^2*Go*L1*L2*r + Cgs^2*L1^2)^(1/2) +
C*L2 + Cgd*L1 + Cgs*L1 + Cgd*L2 + C*Cgs*r^2 + Cgd*Cgs*r^2 + C*Go*L1*r + C*Go*L2*r
+ Cgd*Go*L1*r + Cgs*Go*L1*r + Cgd*Go*L2*r + Cgs*Go*L2*r + C*L1*gm*r + C*L2*gm*r +
Cgd*L1*gm*r + Cgd*L2*gm*r)/(2*C*Cgs*L1*L2 + 2*Cgd*Cgs*L1*L2))^(1/2)
```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Substitute omega into the equation from the imaginary part to select g_m for oscillation.

```
0 = ((C*L1 - (C^2*Cgs^2*r^4 + 2*C^2*Cgs*Go*L1*r^3 + 2*C^2*Cgs*Go*L2*r^3 + 2*C^2*Cgs*L1*gm*r^3
+ 2*C^2*Cgs*L1*r^2 + 2*C^2*Cgs*L2*gm*r^3 + 2*C^2*Cgs*L2*r^2 + C^2*Go^2*L1^2*r^2 +
2*C^2*Go^2*L1*L2*r^2 + C^2*Go^2*L2^2*r^2 + 2*C^2*Go*L1^2*gm*r^2 + 2*C^2*Go*L1^2*r +
4*C^2*Go*L1*L2*gm*r^2 + 4*C^2*Go*L1*L2*r + 2*C^2*Go*L2^2*gm*r^2 + 2*C^2*Go*L2^2*r +
C^2*L1^2*gm^2*r^2 + 2*C^2*L1^2*gm*r + C^2*L1^2 + 2*C^2*L1*L2*gm^2*r^2 + 4*C^2*L1*L2*gm*r +
2*C^2*L1*L2 + C^2*L2^2*gm^2*r^2 + 2*C^2*L2^2*gm*r + C^2*L2^2 + 2*C*Cgd*Cgs^2*r^4 +
4*C*Cgd*Cgs*Go*L1*r^3 + 4*C*Cgd*Cgs*Go*L2*r^3 + 4*C*Cgd*Cgs*L1*gm*r^3 + 4*C*Cgd*Cgs*L1*r^2 +
4*C*Cgd*Cgs*L2*gm*r^3 + 4*C*Cgd*Cgs*L2*r^2 + 2*C*Cgd*Go^2*L1^2*r^2 + 4*C*Cgd*Go^2*L1*L2*r^2 +
2*C*Cgd*Go^2*L2^2*r^2 + 4*C*Cgd*Go*L1^2*gm*r^2 + 4*C*Cgd*Go*L1^2*r + 8*C*Cgd*Go*L1*L2*gm*r^2 +
8*C*Cgd*Go*L1*L2*r + 4*C*Cgd*Go*L2^2*gm*r^2 + 4*C*Cgd*Go*L2^2*r + 2*C*Cgd*L1^2*gm^2*r^2 +
4*C*Cgd*L1^2*gm*r + 2*C*Cgd*L1^2 + 4*C*Cgd*L1*L2*gm^2*r^2 + 8*C*Cgd*L1*L2*gm*r + 4*C*Cgd*L1*L2
+ 2*C*Cgd*L2^2*gm^2*r^2 + 4*C*Cgd*L2^2*gm*r + 2*C*Cgd*L2^2 + 2*C*Cgs^2*Go*L1*r^3 +
2*C*Cgs^2*Go*L2*r^3 + 2*C*Cgs^2*L1*r^2 + 2*C*Cgs*Go^2*L1^2*r^2 + 4*C*Cgs*Go^2*L1*L2*r^2 +
2*C*Cgs*Go^2*L2^2*r^2 + 2*C*Cgs*Go*L1^2*gm*r^2 + 4*C*Cgs*Go*L1^2*r + 4*C*Cgs*Go*L1*L2*gm*r^2 +
2*C*Cgs*Go*L1*L2*r + 2*C*Cgs*Go*L2^2*gm*r^2 + 2*C*Cgs*Go*L2^2*r + 2*C*Cgs*L1^2*gm*r +
2*C*Cgs*L1^2 + 2*C*Cgs*L1*L2*gm*r - 2*C*Cgs*L1*L2 + Cgd^2*Cgs^2*r^4 + 2*Cgd^2*Cgs*Go*L1*r^3 +
2*Cgd^2*Cgs*Go*L2*r^3 + 2*Cgd^2*Cgs*L1*gm*r^3 + 2*Cgd^2*Cgs*L1*r^2 + 2*Cgd^2*Cgs*L2*gm*r^3 +
2*Cgd^2*Cgs*L2*r^2 + Cgd^2*Go^2*L1^2*r^2 + 2*Cgd^2*Go^2*L1*L2*r^2 + Cgd^2*Go^2*L2^2*r^2 +
2*Cgd^2*Go*L1^2*gm*r^2 + 2*Cgd^2*Go*L1^2*r + 4*Cgd^2*Go*L1*L2*gm*r^2 + 4*Cgd^2*Go*L1*L2*r +
2*Cgd^2*Go*L2^2*gm*r^2 + 2*Cgd^2*Go*L2^2*r + Cgd^2*L1^2*gm^2*r^2 + 2*Cgd^2*L1^2*gm*r +
Cgd^2*L1^2 + 2*Cgd^2*L1*L2*gm^2*r^2 + 4*Cgd^2*L1*L2*gm*r + 2*Cgd^2*L1*L2 + Cgd^2*L2^2*gm^2*r^2
+ 2*Cgd^2*L2^2*gm*r + Cgd^2*L2^2 + 2*Cgd*Cgs^2*Go*L1*r^3 + 2*Cgd*Cgs^2*Go*L2*r^3 +
2*Cgd*Cgs^2*L1*r^2 + 2*Cgd*Cgs*Go^2*L1^2*r^2 + 4*Cgd*Cgs*Go^2*L1*L2*r^2 +
2*Cgd*Cgs*Go^2*L2^2*r^2 + 2*Cgd*Cgs*Go*L1^2*gm*r^2 + 4*Cgd*Cgs*Go*L1^2*r +
4*Cgd*Cgs*Go*L1*L2*gm*r^2 + 2*Cgd*Cgs*Go*L1*L2*r + 2*Cgd*Cgs*Go*L2^2*gm*r^2 +
2*Cgd*Cgs*Go*L2^2*r + 2*Cgd*Cgs*L1^2*gm*r + 2*Cgd*Cgs*L1^2 + 2*Cgd*Cgs*L1*L2*gm*r -
2*Cgd*Cgs*L1*L2 + Cgs^2*Go^2*L1^2*r^2 + 2*Cgs^2*Go^2*L1*L2*r^2 + Cgs^2*Go^2*L2^2*r^2 +
2*Cgs^2*Go*L1^2*r + 2*Cgs^2*Go*L1*L2*r + Cgs^2*L1^2)^(1/2) + C*L2 + Cgd*L1 + Cgs*L1 + Cgd*L2 +
C*Cgs*r^2 + Cgd*Cgs*r^2 + C*Go*L1*r + C*Go*L2*r + Cgd*Go*L1*r + Cgs*Go*L1*r + Cgd*Go*L2*r +
Cgs*Go*L2*r + C*L1*gm*r + C*L2*gm*r + Cgd*L1*gm*r + Cgd*L2*gm*r)/(2*C*Cgs*L1*L2 +
2*Cgd*Cgs*L1*L2))^(1/2)*(2*C*r - L2*gm + 2*Cgd*r + Cgs*r + L2*(Go + gm) + C*r^2*(Go + gm) +
Cgd*r^2*(Go + gm) + Cgs*r^2*(Go + gm) - Cgs*gm*r^2) - ((C*L1 - (C^2*Cgs^2*r^4 +
2*C^2*Cgs*Go*L1*r^3 + 2*C^2*Cgs*Go*L2*r^3 + 2*C^2*Cgs*L1*gm*r^3 + 2*C^2*Cgs*L1*r^2 +
2*C^2*Cgs*L2*gm*r^3 + 2*C^2*Cgs*L2*r^2 + C^2*Go^2*L1^2*r^2 + 2*C^2*Go^2*L1*L2*r^2 +
C^2*Go^2*L2^2*r^2 + 2*C^2*Go*L1^2*gm*r^2 + 2*C^2*Go*L1^2*r + 4*C^2*Go*L1*L2*gm*r^2 +
4*C^2*Go*L1*L2*r + 2*C^2*Go*L2^2*gm*r^2 + 2*C^2*Go*L2^2*r + C^2*L1^2*gm^2*r^2 +
2*C^2*L1^2*gm*r + C^2*L1^2 + 2*C^2*L1*L2*gm^2*r^2 + 4*C^2*L1*L2*gm*r + 2*C^2*L1*L2 +
C^2*L2^2*gm^2*r^2 + 2*C^2*L2^2*gm*r + C^2*L2^2 + 2*C*Cgd*Cgs^2*r^4 + 4*C*Cgd*Cgs*Go*L1*r^3 +
4*C*Cgd*Cgs*Go*L2*r^3 + 4*C*Cgd*Cgs*L1*gm*r^3 + 4*C*Cgd*Cgs*L1*r^2 + 4*C*Cgd*Cgs*L2*gm*r^3 +
4*C*Cgd*Cgs*L2*r^2 + 2*C*Cgd*Go^2*L1^2*r^2 + 4*C*Cgd*Go^2*L1*L2*r^2 + 2*C*Cgd*Go^2*L2^2*r^2 +
4*C*Cgd*Go*L1^2*gm*r^2 + 4*C*Cgd*Go*L1^2*r + 8*C*Cgd*Go*L1*L2*gm*r^2 + 8*C*Cgd*Go*L1*L2*r +
4*C*Cgd*Go*L2^2*gm*r^2 + 4*C*Cgd*Go*L2^2*r + 2*C*Cgd*L1^2*gm^2*r^2 + 4*C*Cgd*L1^2*gm*r +
2*C*Cgd*L1^2 + 4*C*Cgd*L1*L2*gm^2*r^2 + 8*C*Cgd*L1*L2*gm*r + 4*C*Cgd*L1*L2 +
2*C*Cgd*L2^2*gm^2*r^2 + 4*C*Cgd*L2^2*gm*r + 2*C*Cgd*L2^2 + 2*C*Cgs^2*Go*L1*r^3 +
2*C*Cgs^2*Go*L2*r^3 + 2*C*Cgs^2*L1*r^2 + 2*C*Cgs*Go^2*L1^2*r^2 + 4*C*Cgs*Go^2*L1*L2*r^2 +
2*C*Cgs*Go^2*L2^2*r^2 + 2*C*Cgs*Go*L1^2*gm*r^2 + 4*C*Cgs*Go*L1^2*r + 4*C*Cgs*Go*L1*L2*gm*r^2 +
2*C*Cgs*Go*L1*L2*r + 2*C*Cgs*Go*L2^2*gm*r^2 + 2*C*Cgs*Go*L2^2*r + 2*C*Cgs*L1^2*gm*r +
2*C*Cgs*L1^2 + 2*C*Cgs*L1*L2*gm*r - 2*C*Cgs*L1*L2 + Cgd^2*Cgs^2*r^4 + 2*Cgd^2*Cgs*Go*L1*r^3 +
2*Cgd^2*Cgs*Go*L2*r^3 + 2*Cgd^2*Cgs*L1*gm*r^3 + 2*Cgd^2*Cgs*L1*r^2 + 2*Cgd^2*Cgs*L2*gm*r^3 +
2*Cgd^2*Cgs*L2*r^2 + Cgd^2*Go^2*L1^2*r^2 + 2*Cgd^2*Go^2*L1*L2*r^2 + Cgd^2*Go^2*L2^2*r^2 +
2*Cgd^2*Go*L1^2*gm*r^2 + 2*Cgd^2*Go*L1^2*r + 4*Cgd^2*Go*L1*L2*gm*r^2 + 4*Cgd^2*Go*L1*L2*r +
2*Cgd^2*Go*L2^2*gm*r^2 + 2*Cgd^2*Go*L2^2*r + Cgd^2*L1^2*gm^2*r^2 + 2*Cgd^2*L1^2*gm*r +
Cgd^2*L1^2 + 2*Cgd^2*L1*L2*gm^2*r^2 + 4*Cgd^2*L1*L2*gm*r + 2*Cgd^2*L1*L2 + Cgd^2*L2^2*gm^2*r^2
+ 2*Cgd^2*L2^2*gm*r + Cgd^2*L2^2 + 2*Cgd*Cgs^2*Go*L1*r^3 + 2*Cgd*Cgs^2*Go*L2*r^3 +
2*Cgd*Cgs^2*L1*r^2 + 2*Cgd*Cgs*Go^2*L1^2*r^2 + 4*Cgd*Cgs*Go^2*L1*L2*r^2 +
2*Cgd*Cgs*Go^2*L2^2*r^2 + 2*Cgd*Cgs*Go*L1^2*gm*r^2 + 4*Cgd*Cgs*Go*L1^2*r +
4*Cgd*Cgs*Go*L1*L2*gm*r^2 + 2*Cgd*Cgs*Go*L1*L2*r + 2*Cgd*Cgs*Go*L2^2*gm*r^2 +
2*Cgd*Cgs*Go*L2^2*r + 2*Cgd*Cgs*L1^2*gm*r + 2*Cgd*Cgs*L1^2 + 2*Cgd*Cgs*L1*L2*gm*r -
2*Cgd*Cgs*L1*L2 + Cgs^2*Go^2*L1^2*r^2 + 2*Cgs^2*Go^2*L1*L2*r^2 + Cgs^2*Go^2*L2^2*r^2 +
2*Cgs^2*Go*L1^2*r + 2*Cgs^2*Go*L1*L2*r + Cgs^2*L1^2)^(1/2) + C*L2 + Cgd*L1 + Cgs*L1 + Cgd*L2 +
C*Cgs*r^2 + Cgd*Cgs*r^2 + C*Go*L1*r + C*Go*L2*r + Cgd*Go*L1*r + Cgs*Go*L1*r + Cgd*Go*L2*r +
Cgs*Go*L2*r + C*L1*gm*r + C*L2*gm*r + Cgd*L1*gm*r + Cgd*L2*gm*r)/(2*C*Cgs*L1*L2 +
2*Cgd*Cgs*L1*L2))^(3/2)*(C*L1*L2*(Go + gm) + Cgd*L1*L2*(Go + gm) + Cgs*L1*L2*(Go + gm) -
Cgs*L1*L2*gm + C*Cgs*L1*r + C*Cgs*L2*r + Cgd*Cgs*L1*r + Cgd*Cgs*L2*r)
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