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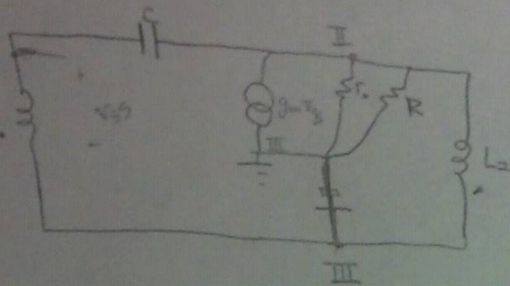
ENEE 610
Problem Set 1.4

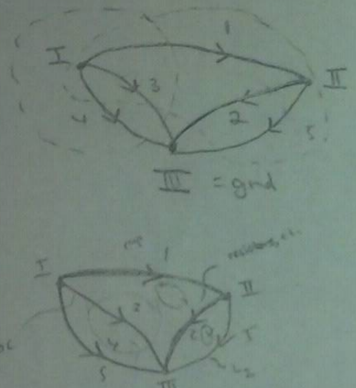
Problem 4

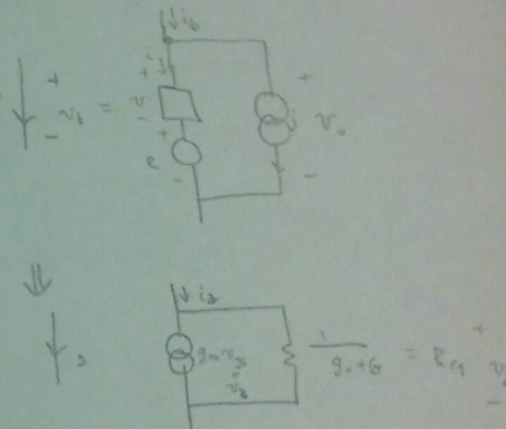


coils coupled w/ coefficient of coupling $\frac{1}{2}$

$$M = k\sqrt{L_1 L_2} = \frac{1}{2}\sqrt{L_1 L_2}$$

branch 1 & 2 are the tree branches





$\frac{1}{g_o + G} = R_{eq}$

$$\left.\begin{aligned} i_b &= i + j \\ v_b &= v + e \end{aligned}\right\} \begin{aligned} i &= i_b - j \\ v &= v_b - e \end{aligned}$$

Cut set Eqns

KCL:

I: $i_1 + i_3 + i_4 = 0$

II: $i_2 + i_3 + i_4 + i_5 = 0$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

1_2

$0 = C i_b$

cutset matrix

Tie set Eqns

KVL:

3: $-V_1 - V_2 + V_3 = 0$

5: $-V_2 + V_5 = 0$

4: $-V_1 - V_2 + V_4 = 0$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}$$

1_3

$0 = T v_b$

tie set matrix

Assume we have linearized around the operating pt. (= Q-bias)

$$s = \frac{d}{dt}$$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

general eqn for inductors 1 and 2

$$\begin{bmatrix} Cs & 0 & 0 & 0 & 0 \\ 0 & g_o+G & g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & L_1 s & Ms \\ 0 & 0 & 0 & Ms & L_2 s \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

Unconnected Eqns

Semi-state Eqns

$$A(s)\, C^T v_t = B(s)\, J^T i_c$$

$$\begin{bmatrix} Cs & 0 & 0 & 0 & 0 \\ 0 & g_o+G & g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{v_t} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & L_1 s & Ms \\ 0 & 0 & 0 & Ms & L_2 s \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} i_3 \\ i_4 \\ i_5 \end{bmatrix}}_{i_c}$$

$$\begin{bmatrix} Cs & 0 \\ g_m & g_o+G+g_m \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & L_1 s & Ms \\ 0 & Ms & L_2 s \end{bmatrix} \begin{bmatrix} i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

$$\left[\begin{array}{cc|ccc} Cs & 0 & 1 & 1 & 0 \\ g_m & g_o+G+g_m & 1 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & -L_1 s & -Ms \\ 0 & 1 & 0 & -Ms & -L_2 s \end{array}\right] x = \underline{0}_5 \qquad \text{where } x = \begin{bmatrix} v_t \\ \hline i_c \end{bmatrix}$$

$$s \begin{bmatrix} C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_1 & -M \\ 0 & 0 & 0 & -M & -L_2 \end{bmatrix} x = \begin{bmatrix} 0 & 0 & -1 & -1 & 0 \\ -g_m & -g_o-G-g_m & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} x$$

Semi-State Eqns

State Eqns

reduce

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eliminate via 3rd row:

$$0 = i_3$$

eliminate via 2nd row:

$$0 = -g_m v_1 - (g_0 + G + g_m) v_2 - \cancel{i_3}^{\,0} - i_4 - i_5$$

$$v_2 = \frac{-i_4 - i_5 - g_m v_1}{g_0 + G + g_m}$$

$$sCv_1 = -\cancel{i_3}^{\,0} - i_4 \quad = -i_4 \quad \Rightarrow sCv_1 = -i_4$$

$$-sL_1 i_4 - sMi_5 = -V_1 - V_2 \quad \Rightarrow -sL_1 i_4 - sMi_5 = -v_1 - \frac{(-i_4 - i_5 - g_m v_1)}{\underbrace{g_0 + G}_{G_0} + g_m}$$

$$-sL_1 i_4 - sMi_5 = -v_1 + \frac{i_4 + i_5 + g_m v_1}{G_0 + g_m}$$

$$= \frac{-v_1(G_0 + g_m) + i_4 + i_5 + g_m v_1}{G_0 + g_m}$$

$$= \frac{-v_1 G_0 + i_4 + i_5}{G_0 + g_m}$$

$$-sMi_4 - sL_2 i_5 = -V_2$$

$$\Rightarrow -sMi_4 - sL_2 i_5 = \frac{+i_4 + i_5 + g_m v_1}{\underbrace{g_0 + G}_{G_0} + g_m}$$

$$s\underbrace{\begin{bmatrix} C & 0 & 0 \\ 0 & -L_1 & -M \\ 0 & -M & -L_2 \end{bmatrix}}_{E} \underline{x} = \overbrace{\begin{bmatrix} 0 & -1 & 0 \\ -\frac{G_0}{G_0+g_m} & \frac{1}{G_0+g_m} & \frac{1}{G_0+g_m} \\ \frac{g_m}{G_0+g_m} & \frac{1}{G_0+g_m} & \frac{1}{G_0+g_m} \end{bmatrix}}^{A} \underline{x}$$

where $\underline{x} = \begin{bmatrix} v_1 \\ i_4 \\ i_5 \end{bmatrix}$

$$E = \begin{bmatrix} C & 0 & 0 \\ 0 & -L_1 & -M \\ 0 & -M & -L_2 \end{bmatrix} \Rightarrow E^{-1} = \begin{bmatrix} \frac{1}{C} & 0 & 0 \\ 0 & \frac{-L_2}{L_1L_2 - M^2} & \frac{M}{L_1L_2 - M^2} \\ 0 & \frac{M}{L_1L_2 - M^2} & \frac{-L_1}{L_1L_2 - M^2} \end{bmatrix}$$

$$(sE - A)\, X(s) = x(0)$$

$$X(s) = (sE - A)^{-1}\, x(0)$$

$$\det(sE - A) = 0$$

$$\frac{1}{G_0+g_m} \det \begin{bmatrix} sC & G_0+g_m & 0 \\ G_0 & -sL_1-1 & -sM-1 \\ -g_m & -sM-1 & -sL_2-1 \end{bmatrix} = 0$$

$$= \frac{1}{G_0+g_m}\left(sC \det \begin{bmatrix} -sL_1-1 & -sM-1 \\ -sM-1 & -sL_2-1 \end{bmatrix} - (G_0+g_m) \det \begin{bmatrix} G_0 & -sM-1 \\ -g_m & -sL_2-1 \end{bmatrix} \right)$$

$$= \frac{sC}{G_0+g_m}\Big((-sL_1-1)(-sL_2-1) - (-sM-1)(-sM-1)\Big) - \Big(G_0(-sL_2-1) - (-g_m)(-sM-1)\Big)$$

$$= \frac{sC}{G_0+g_m}\Big(\big(s^2L_1L_2 + sL_1 + sL_2 + 1 - s^2M^2 - 2sM - 1\big)\Big) + G_0L_2s + G_0 + g_mMs - 1$$

$$= \frac{s^3C\,L_1L_2}{G_0+g_m} + \frac{s^2CL_1}{G_0+g_m} + \frac{s^2L_2C}{G_0+g_m} - \frac{s^3CM^2}{G_0+g_m} - \frac{2s^2CM}{G_0+g_m} - \frac{sC}{G_0+g_m}$$
$$+ G_0L_2s + G_0 + g_mMs - 1$$

$$= s^3\left(\frac{CL_1L_2}{G_0+g_m} - \frac{CM^2}{G_0+g_m}\right) + s^2\left(\frac{CL_1}{G_0+g_m} + \frac{CL_2}{G_0+g_m} - \frac{2CM}{G_0+g_m}\right) +$$
$$s\left(-\frac{C}{G_0+g_m} + G_0L_2 + g_mM\right) + G_0 - 1$$

$$= s^3\frac{C}{G_0+g_m}\left(L_1L_2 - M^2\right) + s^2\frac{C}{G_0+g_m}\left(L_1 + L_2 - 2M\right) + \ldots$$

given real values for C, G_0, g_m, M the roots will give us the oscillation frequencies