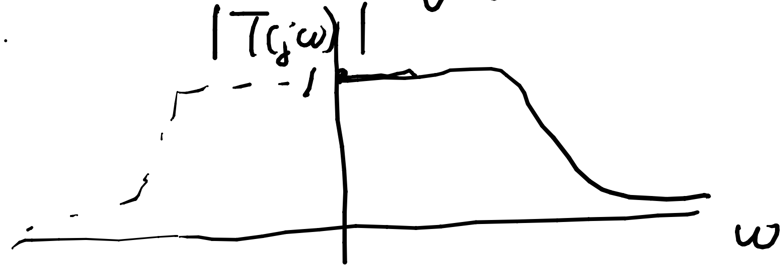
EE610
11/02/10

Maximally flat $T(s)$, low pass



$$T(s) = \frac{K}{s^n + a_{n-1}s^{n-1} + \dots + a_0}$$

normalize to

$$T(s) = \frac{1}{s^n + a_{n-1}s^{n-1} + \dots a_1 s + 1} = \frac{1}{D(s)}$$

$$|T(j\omega)|^2 = \frac{1}{D(j\omega)D(-j\omega)}$$

$$\frac{d|T(j\omega)|^2}{d\omega} = 2|T(j\omega)|\frac{d|T(j\omega)|}{d\omega}$$

$$g(\omega), \quad \frac{d\, 1/g(\omega)}{d\omega} = \frac{dg}{d\omega} \cdot \frac{-1}{g(\omega)^2}$$

∴ look at $\frac{1}{|T(j\omega)|^2}$ & its derivatives:

$$1/|T(j\omega)|^2 = 1 + c_2\omega^2 + \ldots + \omega^{2n}$$

↑ coefficients are the derivatives &

for maximally flat $T(s)$ we set all the coefficients possible to zero

$$|T(j\omega)|^2 = \frac{1}{\omega^{2n}+1} = T(j\omega)\,T(-j\omega)$$

analytically continue by $\omega = s/j$

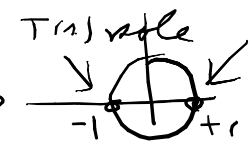
$$T(s)\,T(-s) = \frac{1}{(-1)^n s^{2n} + 1}, \quad \text{poles at } s^{2n} = (-1)/(-1)^n = (-1)^{n+1}$$

$$= \begin{cases} +1 & \text{if } n \text{ odd} \\ -1 & \text{if } n \text{ even} \end{cases}$$

$$\therefore s^{2n} = \begin{cases} e^{j(0+2k\pi)} & n = \text{odd} \\ e^{j(\pi+2k\pi)} & n = \text{even} \end{cases} \quad k = 0, 1, 2, \ldots \bmod(2n)$$

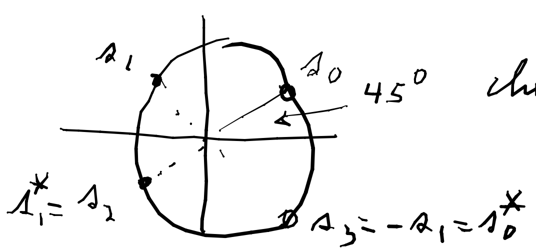
$$\therefore s_k = \begin{cases} e^{j\left(\frac{2k\pi}{2n}\right)} & n \text{ odd} \\ e^{j\left(\frac{\pi+2k\pi}{2n}\right)} & n \text{ even} \end{cases} \quad k = 0, 1, 2, \ldots, 2n-1$$

Ex: $n = 1 = \text{odd} \quad e^{j0} = 1, \; e^{j\frac{2\pi}{2}} = e^{j\pi} = -1 \Rightarrow$



$$\therefore T(s) = \frac{1}{s-(-1)} = \frac{1}{s+1}$$

Ex: $n = 2 = \text{even}, \; \underset{k=0}{e^{j\pi/4}}, \; \underset{k=1}{e^{j3\pi/4}}, \; \underset{k=2}{e^{j5\pi/4}}, \; \underset{k=3}{e^{j7\pi/4}}, \; \underset{k=4\equiv 0 \bmod 4}{e^{j9\pi/4} = e^{j\pi/4}}$

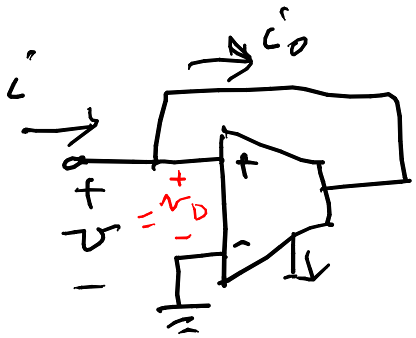


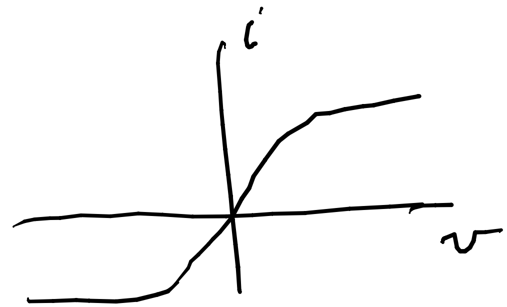
choose LHP poles for $T(s) = \frac{1}{(s-s_1)(s-s_1^*)} = \frac{1}{}$

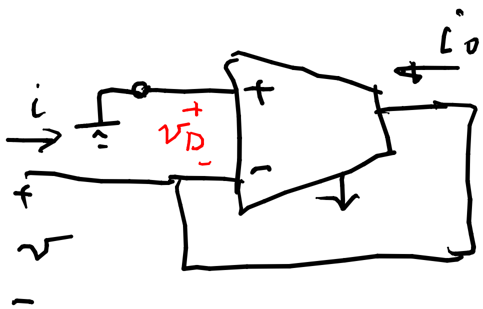
$$= \frac{1}{\left[s - \left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\right]\left[s - \left(-\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\right]} = \frac{1}{s^2 + \sqrt{2}\,s + 1}$$

(Butterworth

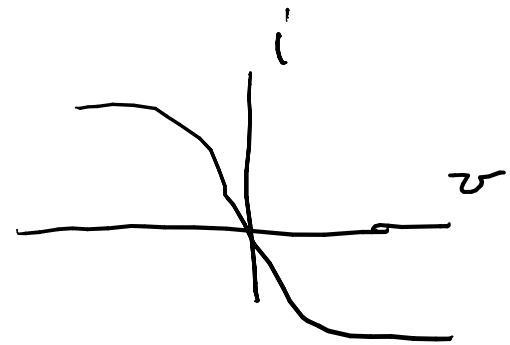
polynomial)





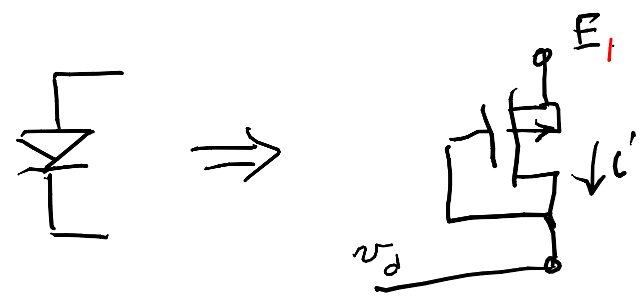


$v_D = -v$

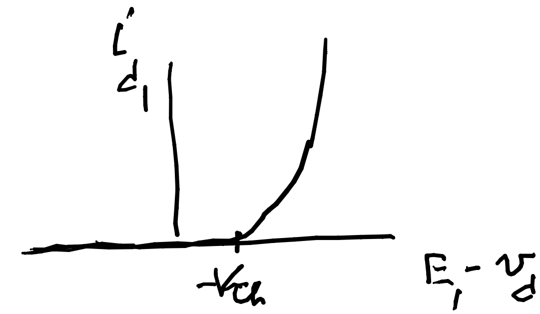


(a negative resistor)

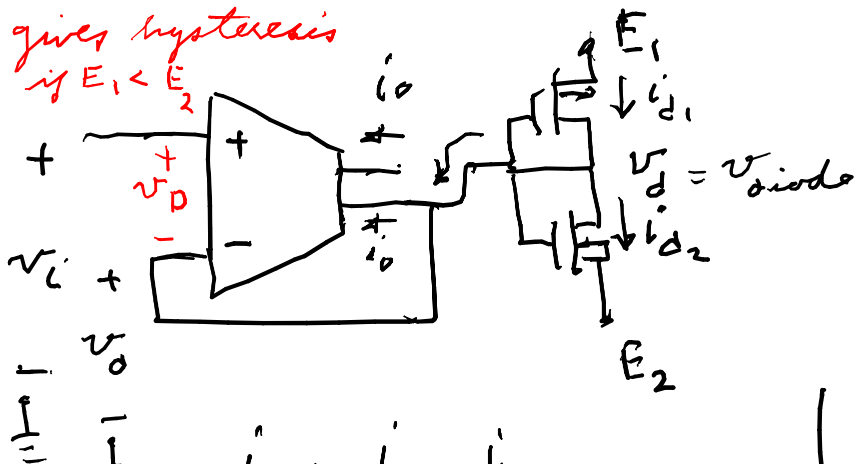
To get hysteresis



E1
i'
vd

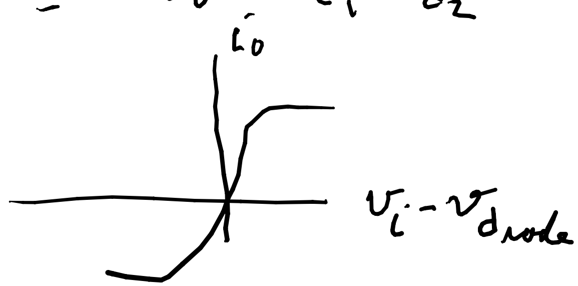


i'd1
Vth
E1 - vd

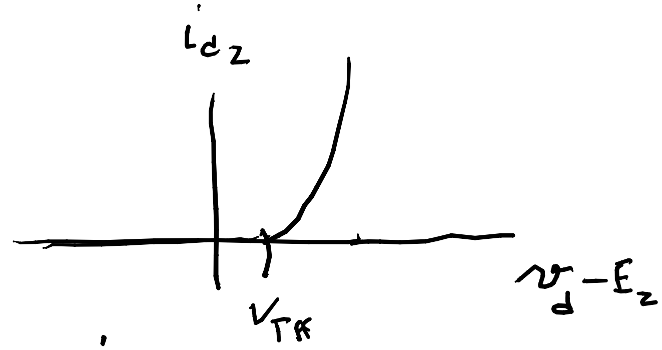


gives hysteresis if E1 < E2
+
vD
vi
vo
io
E1
id1
vd = vdiode
id2
E2

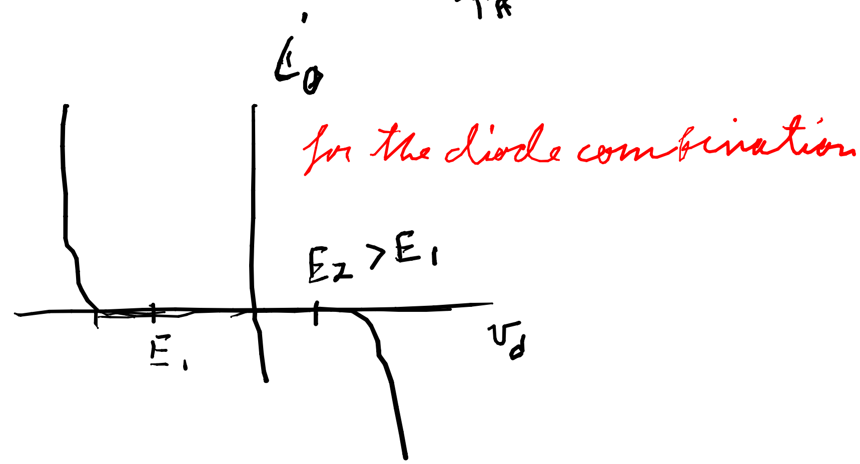
$i_o = i_{d_1} - i_{d_2}$



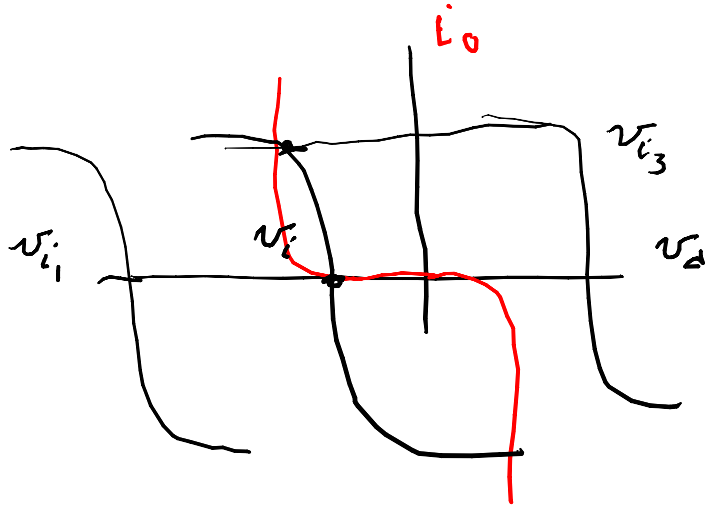
io
vi - vdiode

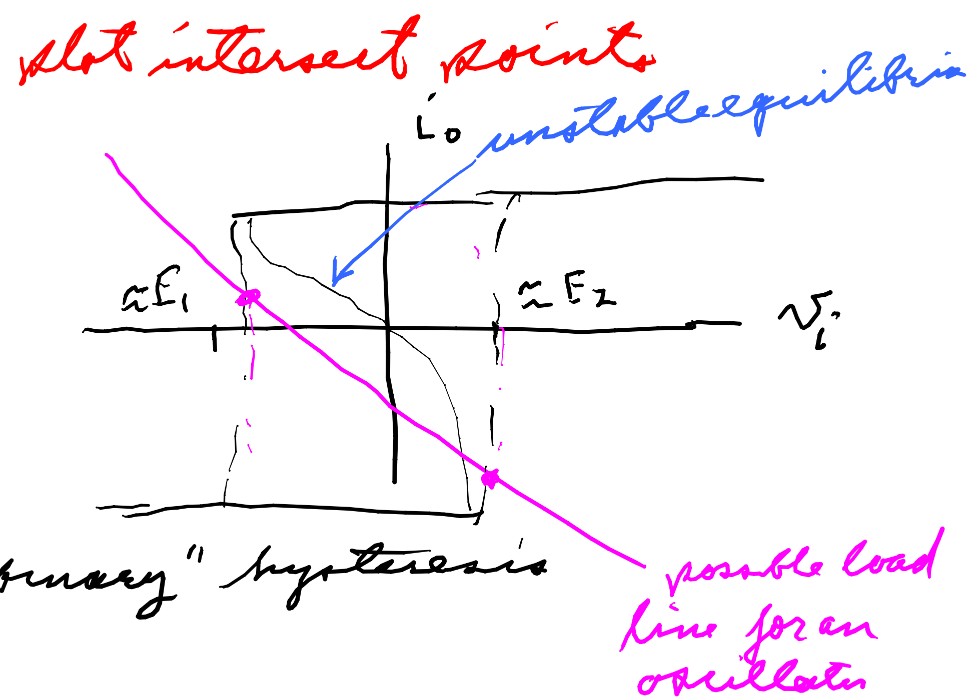


id2
VTR
vd - E2



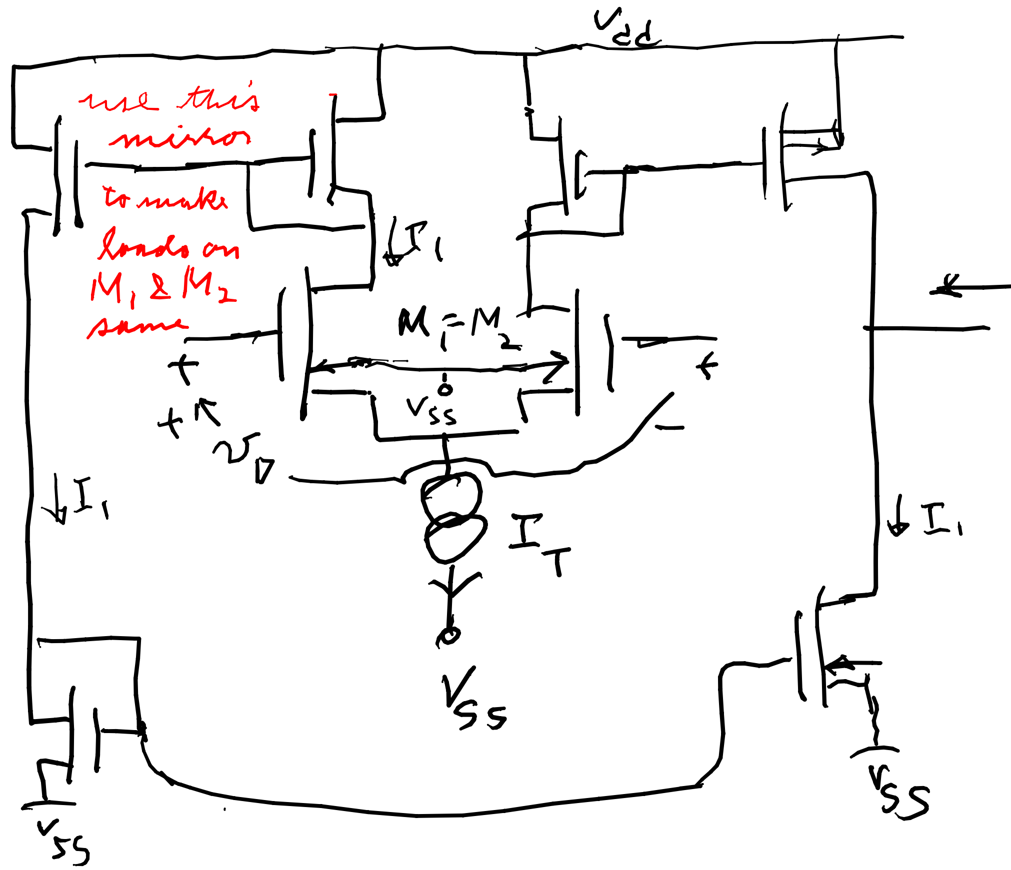
io
for the diode combination
E2 > E1
E1
vd

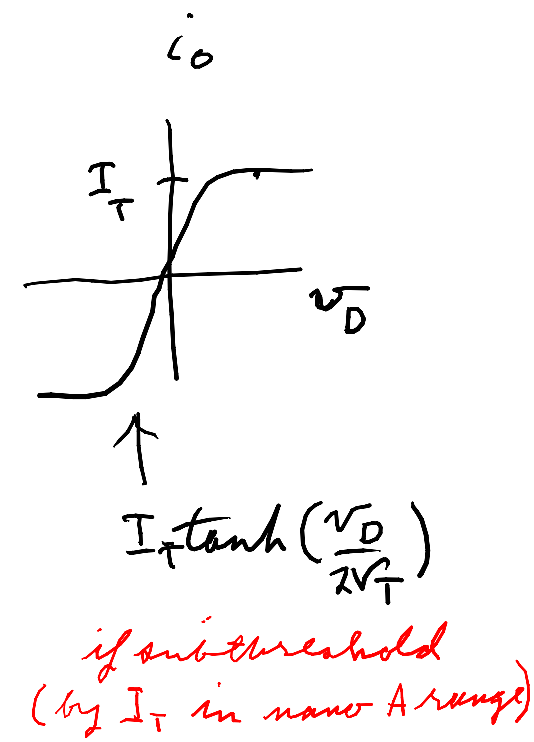




this circuit gives "binary" hysteresis

Making the OTA, via differential pairs

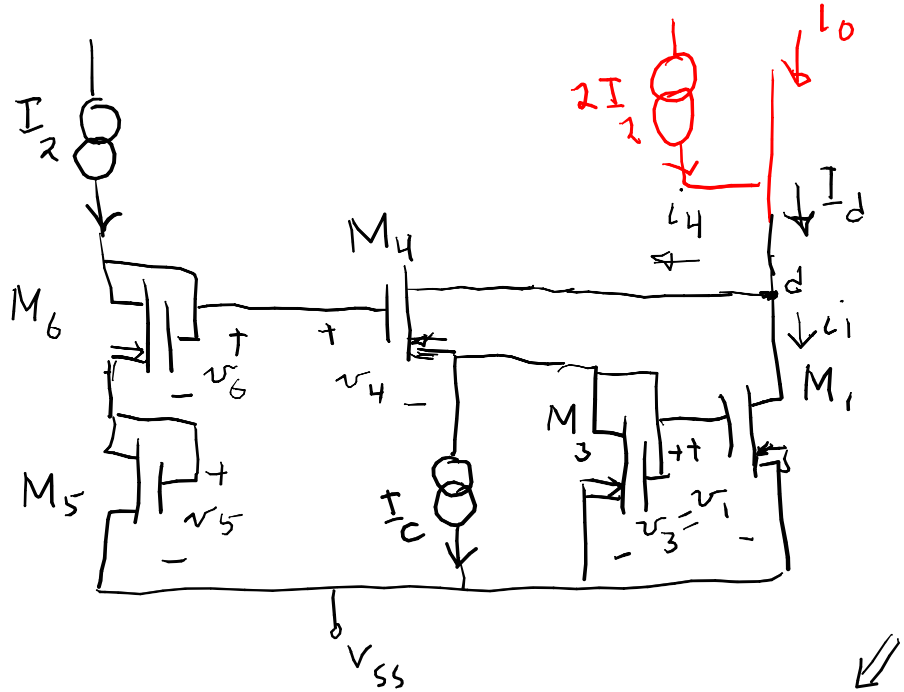




Since M_1 & M_2 have same V_{SB} their threshold voltages are the same & cancel in obtaining i_o

Current square circuit

Takai, et al, CMOS FET Companding Current-mode (IEEE, WT 11-5.1 – 5.4, 1998)



all NMOS transistors identical & in saturation

$$i_D = \beta(v_{GS} - V_{th})^2$$

$$\Rightarrow v_{GS} = V_{th} + \sqrt{i_D/\beta}$$

key equation: $v_i = v_{GS_i}$, $i = 1, \ldots, 6$

$$v_5 + v_6 = v_4 + v_1$$

$\Downarrow$ $i_4 = I_d - i_1 = I_c + i_1$

equivalent $\sqrt{I_2} + \sqrt{I_2} = \sqrt{(I_d - i_1)} + \sqrt{i_1}$

will give a square

out $= I_d - 2I_2 = \left(\frac{1}{8I_2}\right) I_c^2 = \text{input}^2/8I_2$

Squaring $4I_2 = I_d - i_1 + 2\sqrt{(I_d - i_1)i_1} + i_1 \Rightarrow 4I_2 - I_d = 2\sqrt{(I_d - i_1)i_1}$

Squaring again: $16I_2^2 + I_d^2 - 8I_2 I_d = 4(I_d - i_1)i_1$ but $I_d - i_1 = I_c + i_1 = i_4$

$\Rightarrow 2i_1 = I_d - I_c$

$= 4\left(I_d - \frac{(I_d - I_c)}{2}\right)\frac{(I_d - I_c)}{2}$

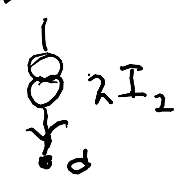
$= (I_d + I_c)(I_d - I_c)$

$= I_d^2 - I_c^2$

cancel I_d^2: $8I_2 I_d = 16I_2^2 + I_c^2$

$\Rightarrow I_d = \frac{1}{8I_2} \cdot I_c^2 + 2I_2$

$\therefore$ if let $i_{out} = i_o = I_d - 2I_2$ by inserting at node d the current source



Then $i_{out} = \left(\frac{1}{8I_2}\right) \cdot i_{in}^2$ if $i_{in} = I_c$

If use $I_d - 2I_2$ as input and an output where I_C is
can also get $i_{out} = \sqrt{8I_2 \cdot i_{in}}$

If the transistors work in subthreshold $i_d = I_0 e^{v_{GS}/V_T}$

then $v_6 + v_5 = v_4 + v_1$ is $2 \ln I_2 = \ln i_4 + \ln i_1$

$$\Rightarrow I_2^2 = i_1 i_4$$

and can restructure to get the square or $\sqrt{\ }$.