EE 610
10/28/10

$$(T_3 T_2 T_1)^{-1} \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -100 \end{bmatrix}}_{Y_d} (T_3 T_2 T_1)^{-T} = Y = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & -1 \\ 4 & -1 & 6 \end{bmatrix};$$

$$(T_3 T_2 T_1)^{-1} = T_1^{-1} T_2^{-1} T_3^{-1} = T, \quad T Y_d T^T = Y$$

$$T_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ +3/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

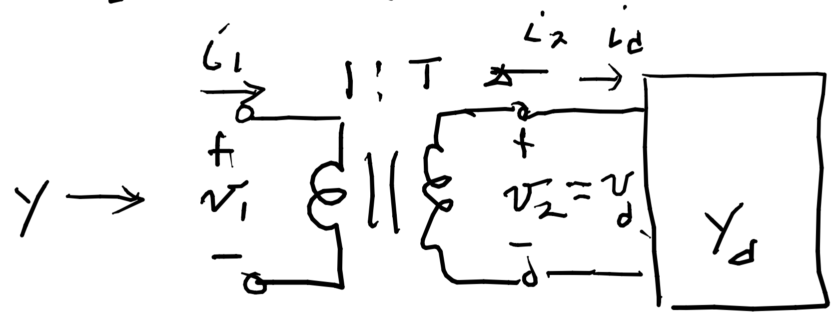
$$\begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = T_1$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}}_{T_2} = T_2$$

$$T_1^{-1} T_2^{-1} T_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -14 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7/1/2 & 1 \end{bmatrix} = T_3$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -14 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 2 & -14 & 1 \end{bmatrix}$$

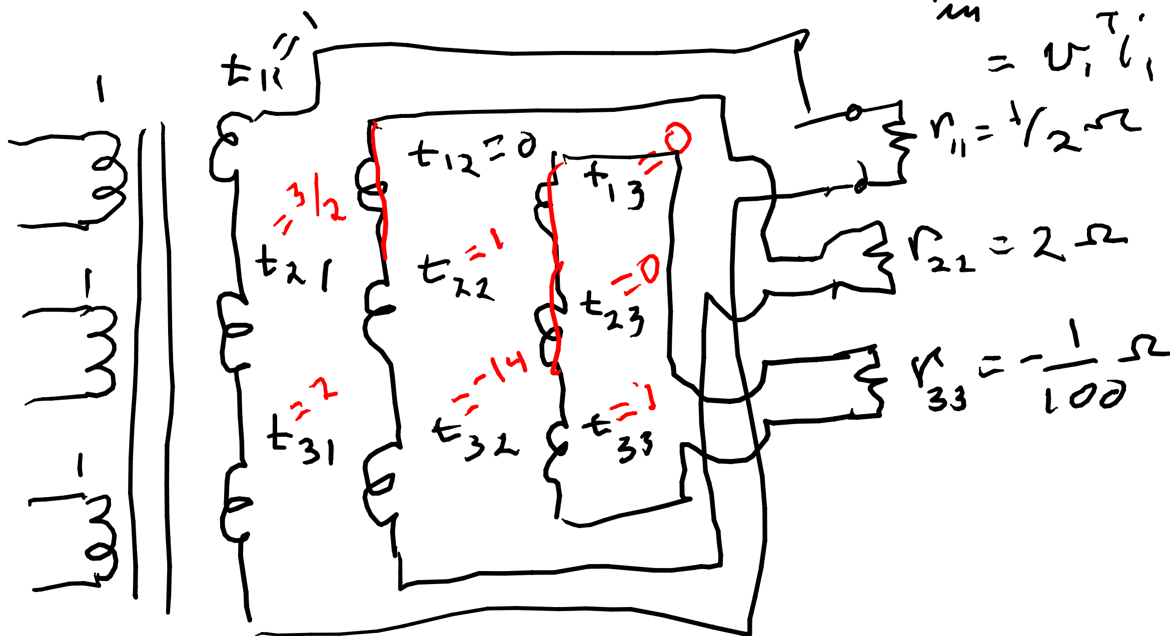
$$Y = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & -1 \\ 4 & -1 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 2 & -14 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -100 \end{bmatrix} \begin{bmatrix} 1 & 3/2 & 2 \\ 0 & 1 & -14 \\ 0 & 0 & 1 \end{bmatrix}$$



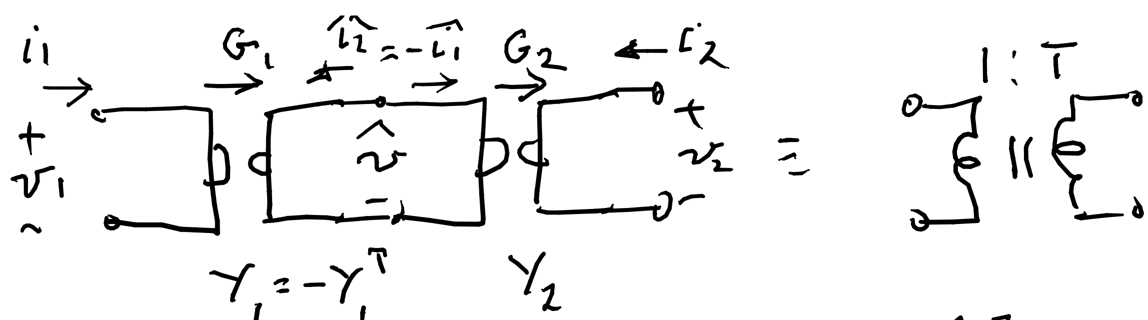
$v_2 = T^T v_1 \quad ; \quad i_1 + T i_2 = 0$

transformer

$P_{in} = v^T i = v_1^T i_1 + v_2^T i_2$
$= v_1^T i_1 + v_1^T T i_2 = 0$



# of gyrators to make a $n \times n$ port transformer in general will use the # of entries in $G_1 + G_2$ given below each is $n \times n$ $\Rightarrow$ $2n^2$ gyrators



0 is $n \times n$

$$\begin{bmatrix} i_1 \\ \hat{i}_2 \end{bmatrix} = \begin{bmatrix} 0 & G_1 \\ -G_1^T & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ \hat{v} \end{bmatrix}, \quad \begin{bmatrix} \hat{i}_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & G_2 \\ -G_2^T & 0 \end{bmatrix} \begin{bmatrix} \hat{v} \\ v_2 \end{bmatrix}$$

$$\hat{i}_2 = -G_1^T v_1 = -\hat{i}_1 = -G_2 v_2 \Rightarrow v_2 = G_2^{-1} G_1^T v_1$$

$$\Rightarrow T^T = G_2^{-1} G_1^T$$

If output $y = Du$ how to get from state equations to semistate equations

$$E\dot{x} = Ax + Bu$$
$$y = Cx$$

$$\underset{\overset{\shortparallel}{0}}{E} \, [\dot{x}] = -1x + Du$$
$$y = Cx$$

$\therefore$ set $E = 0$, $C = 1$, $B = D$ $\qquad 0 \cdot \dot{x} = -1x + Du$

$y = 1x$

How to handle

$y = du/dt \qquad Y(s) = s$ doesn't fit state variable equation

$\dot{x} = Ax + Bu$

$y = Cx + Du$

will fit into semistate equation

(singular, descriptor, differential algebraic)

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

; need $\begin{bmatrix} sE - A \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 \\ s & -1 \end{bmatrix}^{-1}$

$$T(s) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \cdot \frac{1}{1} \begin{bmatrix} -1 & 0 \\ -s & -1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

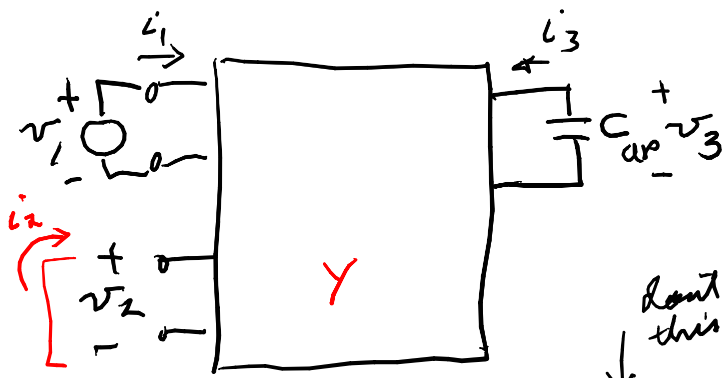
choose $c_1 = b_2 = 0 \Rightarrow T(s) = -c_2 b_1 s$

choose $c_2 = -1$, $b_2 = 1 \Rightarrow T(s) = s$, $\quad y = \frac{du}{dt}$

$$\left.\begin{aligned} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \dot{x} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & -1 \end{bmatrix} x \end{aligned}\right\} \text{gives the differentiator}$$

can get any higher order derivatives by choosing

$$E' = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\dot{x} = Ax + Bu \qquad u = v_1$$

$$y = Cx + Du \qquad y = v_2 \rightarrow i_2$$

don't care this column as multiply by 0

n = dimension of state

don't care $\rightarrow$

$$y = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \begin{matrix} = u \\ = 0 \\ = x \end{matrix}$$

$$-C_{ap}\dot{x} = -C_{ap}\dot{v}_3$$

$\bullet$ = don't care

$$\begin{bmatrix} \bullet \\ y \\ -C_{ap}\dot{x} \end{bmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet \\ D & \bullet & C \\ -B & \bullet & -A \end{bmatrix} \begin{bmatrix} u \\ 0 \\ x \end{bmatrix} \Rightarrow \text{coupling } Y$$

choose $C_{ap} = 1_n$

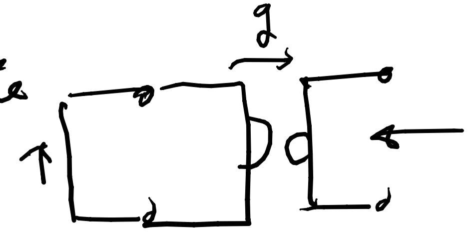
$$Y = \begin{bmatrix} \bullet & \bullet & \bullet \\ D & \bullet & C \\ -B & \bullet & -A \end{bmatrix}$$

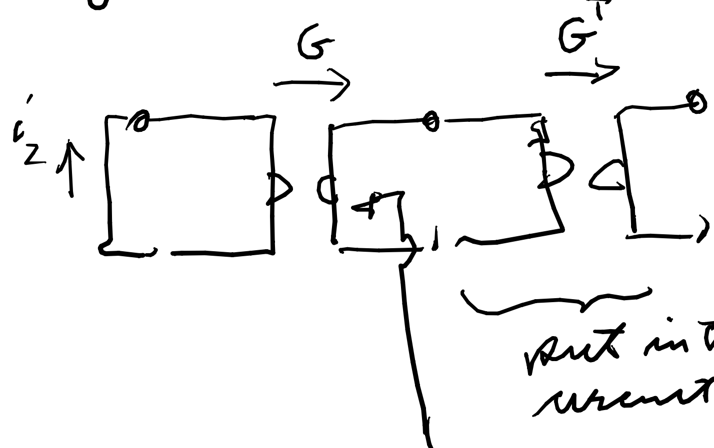
is a constant

∴ fill in ∘ as convenient & synthesize

Y as we did earlier.

But if desire $v_2 \neq 0$ then note





put in the circuit

the circuit sees this = an open circuit

also gives $y = V_2$, $\alpha = V_1$