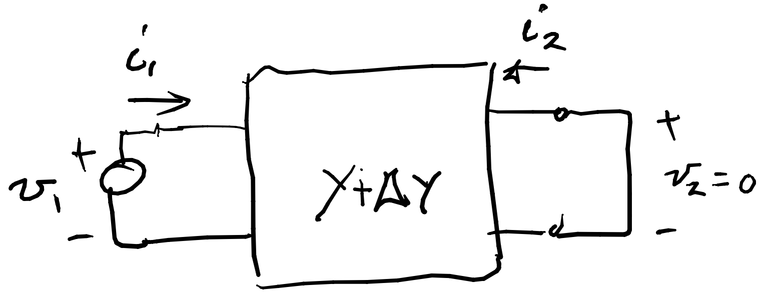
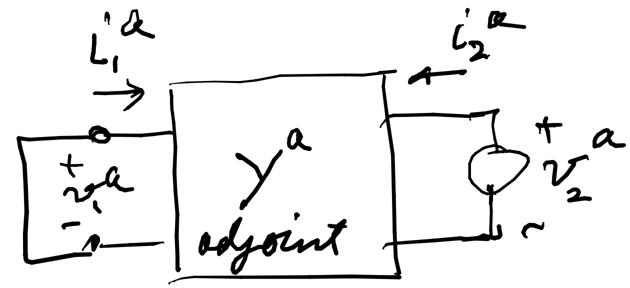
EE 610
10/26/10 ⇒ 10/27/10





here will get
$Y^a = Y^T$

assume the same graph for each

$v_b^T i_b = 0 \qquad v_b^{aT} i_b^a = 0$

$v_b^{aT} i_b - v_b^T i_b^a = 0$ as the same

$$\underbrace{v_{b_1}^a}_{=0} i_{b_1} + \underbrace{v_{b_2}^a}_{=v_2^a} i_{b_2} + \hat{v}_b^{aT} \hat{i}_b - \underbrace{v_{b_1}}_{=v_1} i_{b_1}^a - \underbrace{v_{b_2}}_{=0} i_{b_2}^a - \hat{v}_b^T \hat{i}_b^a$$

d = differential

$$v_2^a i_{b_2} - v_1 i_{b_1}^a + \hat{v}_b^{aT} Y_{b\times b} \hat{v}_b - \underbrace{\hat{v}_b^T Y^a_{b\times b} \hat{v}_b^a}_{= \left(\hat{v}_b^T Y^a_{b\times b} \hat{v}_b^a\right)^T = \hat{v}_b^{aT} Y^{aT}_{b\times b} \hat{v}_b} = 0$$

$$\underset{=0}{(d v_2^a)} i_{b_2} + v_2^a d i_{b_2} - \underset{=0}{(d v_1)} i_{b_1}^a - v_1 \underset{=0}{d i_{b_1}^a}$$

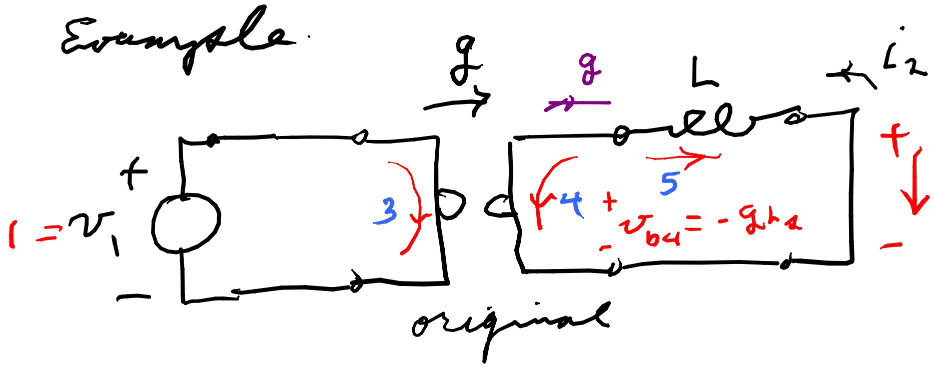
$$+ d\hat{v}_b^{aT} \cdot Y_{b\times b} \hat{v}_b + \hat{v}_b^{aT} dY_{b\times b} \cdot \hat{v}_b + \hat{v}_b^T Y_{b\times b} d\hat{v}_b$$

$$- d\hat{v}_b^{aT} \cdot Y^{aT}_{b\times b} \hat{v}_b - \underset{=0}{\hat{v}_b^T dY^{aT}_{b\times b} \cdot \hat{v}_b} - \hat{v}_b^{aT} Y^{aT}_{b\times b} d\hat{v}_b = 0$$

0 as don't make changes in adjoint circuit

$$v_2^a \cdot d i_{b_2} = v_b^{aT} dY_{b\times b} \cdot \hat{v}_b$$

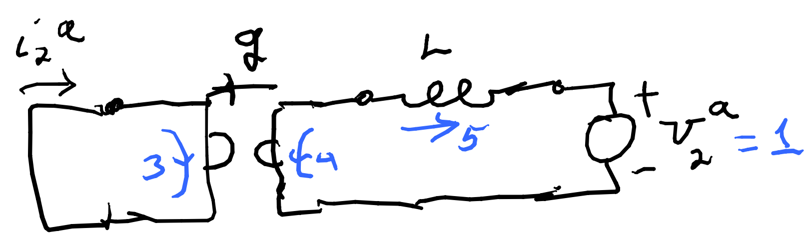
Example.



$$T(s) = \frac{i_2}{v_1}$$

desire $\dfrac{d\,T(s)}{dg}$

$$Y_{b\times b} = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & \frac{1}{Ls} \end{bmatrix}$$



$$Y^a_{b\times b} = \begin{bmatrix} 0 & -g & 0 \\ g & 0 & 0 \\ 0 & 0 & 1/Ls \end{bmatrix}$$

$$\frac{dY_{b\times b}}{dq} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\hat{v}_b^{aT} \frac{dY_{b\times b}}{dq} \hat{v}_b = \begin{bmatrix} v_{b_3}^a & v_{b_4}^a & v_{b_5}^a \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{b_3} \\ v_{b_4} \\ v_{b_5} \end{bmatrix}$$

$$= v_{b_3}^a v_{b_4} - v_{b_4}^a v_{b_3}$$

$$\frac{d\,(-i_2)}{dt} = \frac{d\,i_{b_2}}{dg} = v_{b_3}^a v_{b_4} - v_{b_4}^a v_{b_3} \quad = v_1 = 1 \quad \text{as } v$$

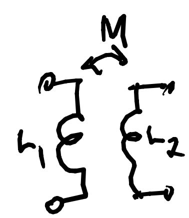
$\therefore$ solve the 2 circuits: (used in chap. 9 section 3)

$$\frac{d\, i_{b_2}}{dq} = v^{\alpha}_{b_3} v_{b_4} - v^{\alpha}_{b_4} v_{b_3}$$

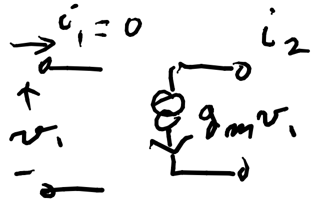
$$= \underbrace{v^{\alpha}_{b_3}}_{=0} (-gLa) - \underbrace{v^{\alpha}_{b_4}}_{=1} \cdot 1 = \frac{d\, i_{b_2}}{dq} = -1$$

from the circuit $i_{b_2} = g$, $-\dfrac{d\, i_{b_2}}{dq} = 1$

Some adjoints

 $Y = \frac{1}{R} = Y^{\alpha}$  $CA = Y = Y^{\alpha}$ 

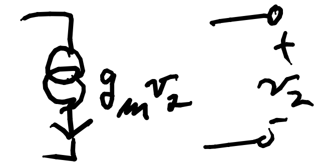
is its own adjoint

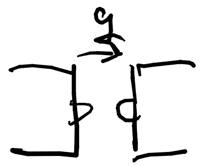


VCCS

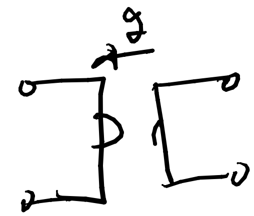
OTA

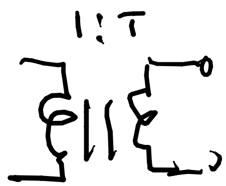
$$Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \Rightarrow Y^a = Y^T = \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix}$$





$$Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \Rightarrow Y^a = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \Rightarrow$$





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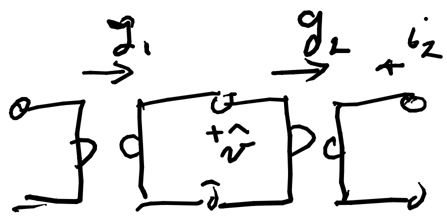

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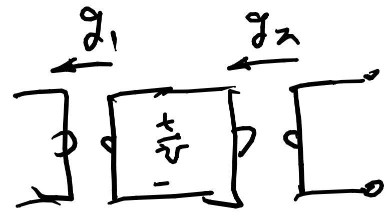

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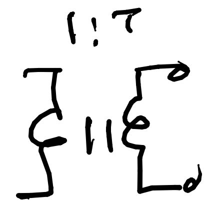


$i_1 + T i_2 = 0$

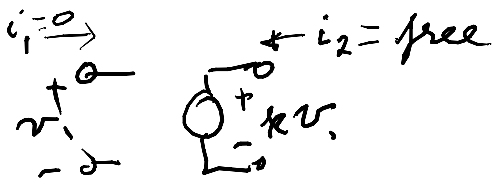
$i_1 = g_1 \hat{v}$; $i_2 = -g_2 \hat{v}$

$\frac{i_1}{i_2} = \frac{-g_1}{g_2} \Rightarrow i_1 + \frac{g_1}{g_2} i_2 = 0$

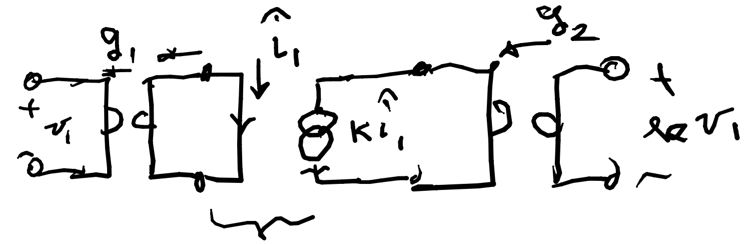
$i_1 = -g_1 \hat{v}$, $i_2 = +g_2 \hat{v}$

$i_1 + \frac{g_1}{g_2} i_2 = 0$

adjoint



VCVS



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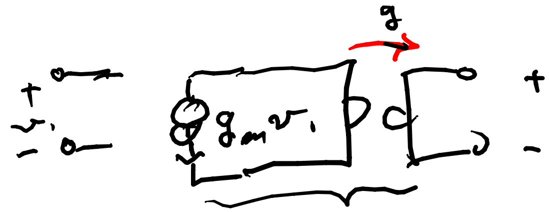
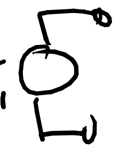
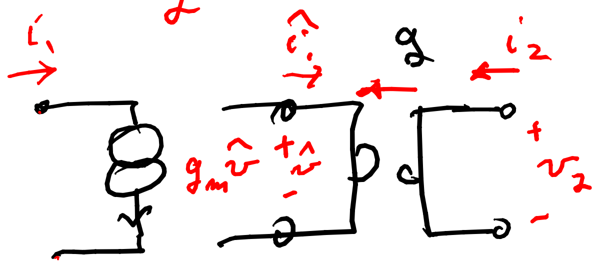


$v_2 = +\frac{1}{g} i_1 = \frac{1}{g}(-g_m v_1) = -\frac{g_m}{g} v_1$

$-\frac{g_m}{g} = kv_1$



adjoint

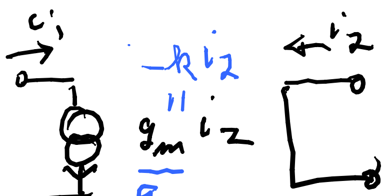


a short

$\hat{i}_1 = -g v_2 = 0 \Rightarrow v_2 = 0$

$i_1 = g_m \hat{v} = g_m\left(-\left(-\frac{1}{g} i_2\right)\right)$

$= \frac{g_m}{g} i_2$



CCCS

Look at synthesizing a symmetric constant Y

$$Y = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & -1 \\ 4 & -1 & 6 \end{bmatrix} ; \quad \underbrace{2}_{=\Delta_{23,23}}, \quad 2\times5-3\times3 = \underbrace{1}_{=\Delta_{33}},$$

$$\Delta = 2\begin{vmatrix} 5 & -1 \\ -1 & 6 \end{vmatrix} - 3\begin{vmatrix} 3 & 4 \\ -1 & 6 \end{vmatrix} + 4\begin{vmatrix} 3 & 4 \\ 5 & -1 \end{vmatrix}$$

$$= 2\times29 - 3\times14 + 4(-23) < 0$$

$\therefore$ Y is not PR

TYT^T;

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_1} \underbrace{\begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & -1 \\ 4 & -1 & 6 \end{bmatrix}}_{Y} \underbrace{\begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_1^T}$$

use T_1 to add $-\frac{y_{21}}{y_{11}} \times$ 1st row to 2nd row (& same for columns by T_1^T)

$$= \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1/2 & -7 \\ 4 & -1 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 1/2 & -7 \\ 4 & -7 & 6 \end{bmatrix}$$

use T_2

to make $(3,1)$ entry 0

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}}_{T_2} \begin{bmatrix} 2 & 0 & 4 \\ 0 & 1/2 & -7 \\ 4 & -7 & 6 \end{bmatrix} \underset{T_2^T}{\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 2 & 0 & 4 \\ 0 & 1/2 & -7 \\ 0 & -7 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1/2 & -7 \\ 0 & -7 & -2 \end{bmatrix}$$

use T_3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7/\tfrac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1/2 & -7 \\ 0 & -7 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 14 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow (3,3)$ entry $= -49 \times 2 - 2 = -100$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1/2 & -7 \\ 0 & 0 & -100 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 7/2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -100 \end{bmatrix}$$

$$= (T_3 T_2 T_1) Y (T_3 T_2 T_1)^T$$