EE 610
10/21/10

$$\begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} A^TQ + QA & QB - C^T \\ B^TQ - C & -D - D^T \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \leq 0$$

$$\hat{Y}_c = \begin{bmatrix} D & \hat{C} \\ -\hat{B} & -\hat{A} \end{bmatrix} = \begin{bmatrix} D & CP^{-1} \\ -PB & -PAP^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & P^{-1} \end{bmatrix}$$

$$Y_c = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} = \left[\begin{array}{c|cc} 1 & 0 & 2 \\ \hline 0 & 0 & -1 \\ -1 & 2 & 1 \end{array}\right]$$

$Q^T = Q$ is positive definite

$$\dot{x} = Ax, \quad x = e^{At}\underbrace{x(0)}_{x_0}$$

$$V(x) = x^T Q x > 0$$
$$= x_0^T e^{A^T t} Q e^{At} x_0$$

$$\dot{V}(x) = \dot{x}^T Q x + x^T Q \dot{x}$$
$$= x_0^T e^{At^T} A^T Q e^{At} x_0$$
$$+ x_0 e^{A^T} Q A e^{At} x_0 \leq 0$$

$$Q = Q_1^T Q_1$$

PR Lemma is that there is a $Q > 0$ and an L & $W \geq 0$ such that

$$\begin{bmatrix} A^T Q + QA & QB - C^T \\ B^T Q - C & -D - D^T \end{bmatrix} = -\begin{bmatrix} L^T L & L^T W \\ W^T L & W^T W + W_0 \end{bmatrix}, \quad W_0 \geq 0$$

$$= -\begin{bmatrix} L^T \\ W^T \end{bmatrix} \begin{bmatrix} L & W \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & W_0 \end{bmatrix}$$

$$Y_c = \begin{bmatrix} D & CP^{-1} \\ -PB & -PAP^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & P^{-1} \end{bmatrix}$$

$$Y_c + Y_c^T = \begin{bmatrix} D + D^T & CP^{-1} - \quad CP^{-1} - B^T P^T \\ -PB + P^{-T} C^T & -P^{-T} A^T P^T - PAP^{-1} \end{bmatrix}$$

$$\begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} A^T Q_1^T Q_1 + Q_1^T Q_1 A & Q_1^T Q_1 B - C^T \\ B^T Q_1^T Q_1 - C & - D - D^T \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \leq 0$$

$$\begin{bmatrix} x^T Q_1^T & u^T \end{bmatrix} \begin{bmatrix} Q_1^{-T} A^T Q_1^T + Q_1 A Q_1^{-1} & Q_1 B - Q_1^{-T} C^T \\ B^T Q_1^T - C Q_1^{-1} & -(D + D^T) \end{bmatrix} \begin{bmatrix} Q_1 x \\ u \end{bmatrix} \leq 0$$

$P = Q_1$ then $Y_c + Y_c^T$ is positive semi definite

$$Y_c = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} = \left[\begin{array}{c|cc} 1 & 0 & 2 \\ \hline 0 & 0 & -1 \\ -1 & 2 & 1 \end{array}\right] \text{ goes to } \tilde{Y}_c = \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix}\begin{bmatrix} D & C \\ -B & -A \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & P^{-1} \end{bmatrix}$$

$$\tilde{Y}_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & p_{11} & p_{12} \\ 0 & p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ -1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & p_{22} & -p_{12} \\ 0 & -p_{12} & p_{11} \end{bmatrix}\frac{1}{p_{11}p_{22} - p_{12}^2} \Big\} > 0$$

choose $p_{12} = 0$

$$\hat{Y}_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & p_{11} & 0 \\ 0 & 0 & p_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ -1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & p_{22} & 0 \\ 0 & 0 & p_{11} \end{bmatrix}\frac{1}{p_{11}p_{22}} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & -p_{11} \\ -p_{22} & 2p_{22} & p_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/p_{11} & 0 \\ 0 & 0 & 1/p_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2/p_{22} \\ 0 & 0 & -p_{11}/p_{22} \\ -p_{22} & 2p_{22}/p_{11} & 1 \end{bmatrix}$$

$$(\gamma_c + \hat{\gamma}_c) = \begin{bmatrix} 1 & 0 & \frac{2}{\rho_{22}} - \rho_{22} \\ 0 & 0 & \frac{2\rho_{22}}{\rho_{11}} - \rho_{11}/\rho_{22} \\ -\rho_{22} + \frac{2}{\rho_{22}} & \frac{2\rho_{22}}{\rho_{11}} - \frac{\rho_{11}}{\rho_{22}} & 1 \end{bmatrix}$$

choose: $\frac{2\rho_{22}}{\rho_{11}} = \frac{\rho_{11}}{\rho_{22}} \Rightarrow \rho_{11}^2 = 2\rho_{22}^2 \Rightarrow \rho_{11} = \sqrt{2}\rho_{22}$

$(2,3) = (3,2) \geq 0$

$(1,2)$: $\frac{2}{\rho_{22}} - \rho_{22} = \frac{2 - \rho_{22}^2}{\rho_{22}} = 0$ if $\rho_{22} = \sqrt{2} \Rightarrow \rho_{11} = 2$

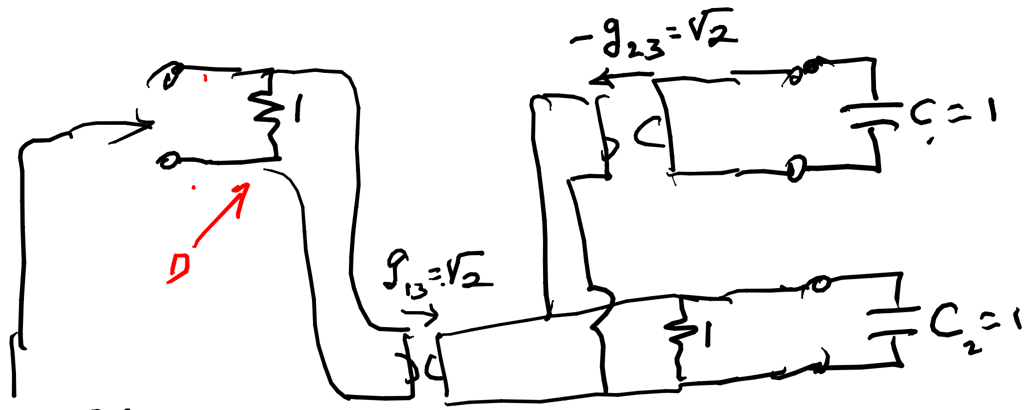
alternative choose $\det \begin{bmatrix} 2 & \frac{-\rho_{22}^2 + 2}{\rho_{22}} \\ \frac{-\rho_{22}^2 + 2}{\rho_{22}} & 2 \end{bmatrix} = \det \begin{bmatrix} (1,1) & (1,3) \\ (3,1) & (3,3) \end{bmatrix}$

$$1 - \left(\frac{-\rho_{22}^2 + 2}{\rho_{22}}\right)^2 = \frac{\rho_{22}^2 - (\rho_{22}^4 - 4\rho_{22}^2 - 4)}{\rho_{22}^2} \geq 0$$

choose $r_{11} = 2$, $r_{22} = \sqrt{2}$ this will give $y_m(s) = \dfrac{2s}{s^2+s+2}$

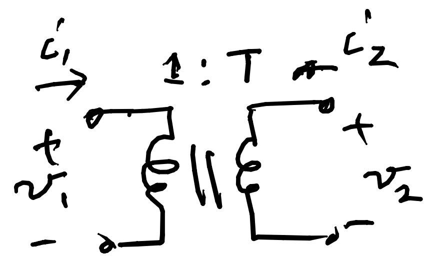
$$\hat{Y}_c = \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 0 & -\sqrt{2} \\ -\sqrt{2} & \sqrt{2} & 1 \end{bmatrix} \text{ is PR}$$

$$\hat{Y}_c = \underbrace{\frac{(Y_c + Y_c^T)}{2}}_{\substack{\text{symmetric} \\ \Uparrow \\ \text{resistors} \\ \text{\& transformers}}} + \underbrace{\frac{(Y_c - Y_c^T)}{2}}_{\substack{\text{skew} \\ \Uparrow \\ \text{gyrators}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \sqrt{2} \\ 0 & 0 & -\sqrt{2} \\ -\sqrt{2} & \sqrt{2} & 0 \end{bmatrix}$$



$$y_{in} = \frac{2s}{s^2+s+2} + 1$$

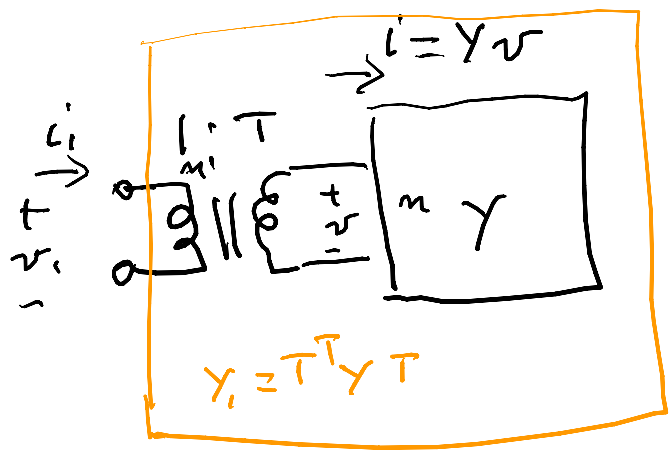
look at transformers



$$p_{in} \equiv 0 = v_1^T i_1 + v_2^T i_2$$

$$v_2 = T v_1 \Rightarrow 0 \quad v_1^T i_1 + v_1^T T^T i_2 = v_1^T \left[i_1 + T^T i_2 \right]$$

$$\Rightarrow \quad i_1 = -T^T i_2$$



$$v = T v_1$$

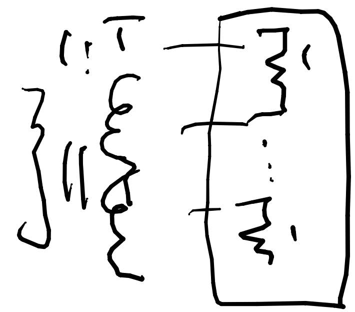
$$i_1 = -T^T(-i)$$

$$= +T^T Y v$$

$$= (T^T Y T) v$$

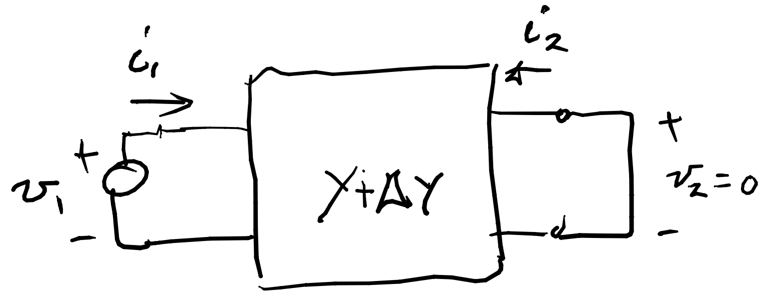
use the T to diagonalize

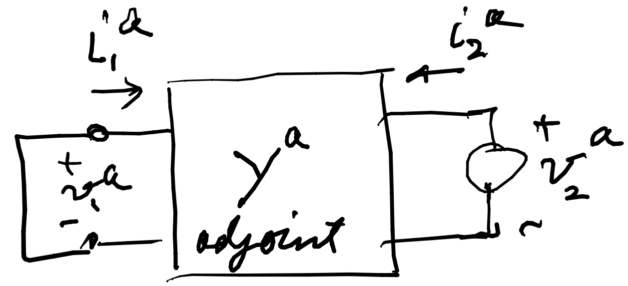
$$T^T\left(\frac{\hat{Y}_c + \hat{Y}_c^T}{2}\right)T = T_1^T T_1 \leftarrow$$



$$\left(\frac{\hat{Y}_c + \hat{Y}_c}{2}\right) = T^{-T}\left(T_1^T \, 1 \, \underbrace{T_1}\right)T^{-1}$$

transformers on unit resistors





here will get $Y^a = Y^T$

assume the same graph for each

$v_b^T i_b = 0 \qquad v_b^{aT} i_b^a = 0$

$v_b^{aT} i_b - v_b^T i_b^a = 0$ as the same