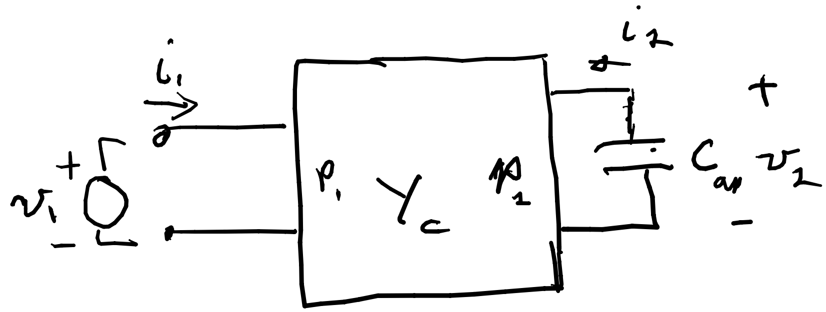
EE 610

10/19/10

revised 10/21/10



$i_1 = T(s)\, v_1$

$\dot{x} = Ax + Bu$

$y = Cx + Du + k_\infty \dot{u}$

$T(s) = D + C(s\mathbb{1}_{p_2} - A)^{-1} B$

$D = T(\infty)$ assuming no poles at ∞

(remove poles at ∞ separately)

k_∞ is positive semidefinite if $T(s)$ is PR

$$y = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{x}$$

$$-C_{ap}\dot{x} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

For homework handed in please put your name, date and course

Normalize $C_{cap} = 1_{\rho_2}$

$$\dot{x} = -y_{21} v_1 - y_{22} x = -y_{22} x - y_{21} v_1 \quad , \quad v_1 = u$$

$$y = i_1 = y_{12} x + y_{11} v_1$$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

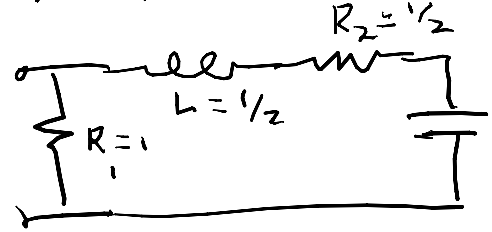
$$Y_c = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix}$$ can make with OTA's

if $T(s)$ is PR desire this Y_c to be PR
to guarantee Y_c to be PR use the PR Lemma

$$Y(s) = T(s) = y(s) = \frac{s^2+3s+2}{s^2+s+2} \; ; \quad D = T(\infty) = 1$$

$$y(s) - 1 = \frac{s^2+3s+2}{s^2+s+2} - 1 = \frac{2s}{s^2+s+2} = \frac{1}{\frac{s}{2} + \frac{1}{2} + \frac{1}{s}} \Big\} = z_1$$

$y(s) \Rightarrow$



this is passive

$$\therefore \quad y(s) = \frac{s^2+3s+2}{s^2+s+2} \text{ is pr} \quad ; \quad \delta(y(s)) = 2 \text{ this is the smallest dimension for } x$$

$$y_1(s) = \frac{2s}{s^2+s+2}$$

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 2 \end{bmatrix} x + [1] u$$

$\Big\}$ gives $y = \frac{s^2+3s+2}{s^2+s+2} \cdot u$

denominator is $\det\begin{bmatrix} s-0 & -1 \\ +2 & s-(-1) \end{bmatrix} = s^2+s+2$

$$T(s) = 1 + \begin{bmatrix} 0 & 2 \end{bmatrix} \frac{1}{s^2+s+2} \begin{bmatrix} s+1 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = D + C(s\mathbb{1}_2 - A)^{-1}B$$

$$= 1 + \frac{1}{s^2+s+2} \begin{bmatrix} -4 & 2s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 + \frac{2s}{s^2+s+2} \quad \text{ok check}$$

$$Y_c = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} = \left[\begin{array}{c|cc} 1 & 0 & 2 \\ \hline 0 & 0 & -1 \\ -1 & 2 & 1 \end{array}\right]$$

$$(Y_c + Y_c^T) = 2\,\mathrm{Re}\,Y_c = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ -1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

is $v^{T*}(Y_c + Y_c^{T*})v \geq 0$ for all 2-vectors v $\Rightarrow$ is $\rightarrow$ positive semi-definite

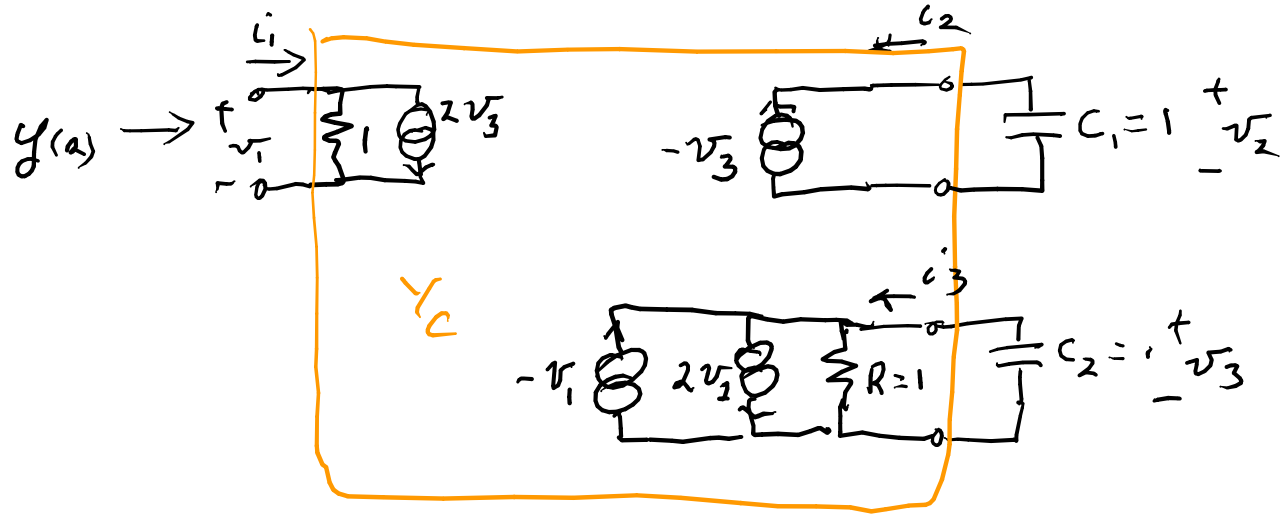
$v^{T*}Y_c + Y_c^{*T}v$

try $v = \begin{bmatrix} 0 \\ v_2 \\ 1 \end{bmatrix}$; $\begin{bmatrix} 0 & v_2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & v_2+2 \end{bmatrix} \begin{bmatrix} 0 \\ v_2 \\ 1 \end{bmatrix} = 2v_2+2$

if $v_2 = -3$, $2v_2+2 = -4 < 0$

means $Y_c + Y_c^T$ is not positive semidefinite

as need all principal minors ≥ 0



$$\dot{x} = Ax + Bu \qquad ; \quad \text{let } \hat{x} = Px$$
$$y = Cx + Du$$

P nonsingular
constant in time
$\dot{} = d/dt$

$$\dot{\hat{x}} = P\dot{x} = PAP^{-1}Px + PBu$$
$$y = CP^{-1}Px + Du$$

$$\dot{\hat{x}} = PAP^{-1}\hat{x} + PBu$$
$$y = CP^{-1}\hat{x} + Du$$

still has $y = T(s)u$

$$T(s) = D + C(s1_{p_2} - A)^{-1}B$$
$$= D + CP^{-1}(s1_{p_2} - PAP^{-1})^{-1}PB$$

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$$
$$y = \hat{C}\hat{x} + Du$$

$$\hat{Y}_c = \begin{bmatrix} D & \hat{C} \\ -\hat{B} & -\hat{A} \end{bmatrix} = \begin{bmatrix} D & CP^{-1} \\ -PB & -PAP^{-1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix}\begin{bmatrix} D & C \\ -B & -A \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & P^{-1} \end{bmatrix}}_{\text{a similarity transformation}}$$

can choose the P to make $\hat{T}_c$ PR

Look at stability via Lyapunov function

$V(x) \geq 0$ like energy

$\dot{V}(x) \leq 0 \longleftarrow$ for stability

$V(x) = x^T Q x$; Q positive definite, constant

$\dot{V}(x) = \dot{x}^T Q x + x^T Q \dot{x} \leq 0$ for a PR $T(s)$

$$= (Ax + Bu)^T Q x + x^T Q (Ax + Bu)$$

$$= x^T A^T Q x + u^T B^T Q x + x^T Q A x + x^T Q B u$$

$$= x^T (A^T Q + QA) x + u^T B^T Q x + x^T Q B u \quad \overset{\text{stability}}{\leq 0}$$

for $u = 0 \Rightarrow \dot{x} = Ax \Rightarrow x(t) = e^{At} x(0)$

$$e^{At} = \sum_{i=0}^{\infty} (At)^i / i! \; ; \; de^{At}/dt = \sum_{i=1}^{\infty} \frac{A\,i\,(At)^{i-1}}{i!} = \sum_{i=1}^{\infty} \frac{(At)^{i-1}}{(i-1)!} = A \sum_{j=0} \frac{(At)^j}{j!}$$

$$\dot V(x) - i^T v - v^T i = \dot V(x) - y^T u - u^T y \le 0$$

passivity

y = output $= y(t)$

$$\Downarrow$$

$$= \dot V(x) - (Cx + Du)^T u - u^T (Cx + Du)$$

$$= \dot V(x) - x^T C^T u - u^T D^T u - u^T C x - u^T D u$$

$$= \begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} A^T Q + QA & QB - C^T \\ B^T Q - C & -\cancel{2}D - D^T \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \le 0$$

$\therefore$ is negative semidefinite if $T(s)$ is PR

(if lossless $D = 0 \Rightarrow QB = C^T$)

The positive real lemma is that there exists a positive definite Q to make this matrix positive semi-definite