EE610

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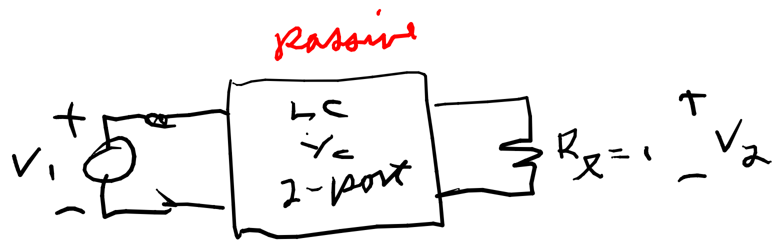


$y_{12} = y_{21}$

reciprocal 2-port

$Y_c = Y_c^T$ $\quad$ T = transpose

assume rational with real coefficients

$$\frac{V_2}{V_1} = \frac{-y_{21}}{1 + y_{22}} = \frac{N(s)}{D(s)}$$

D = Hurwitz

$$y_{21} = k_{21_\infty} s + \frac{k_{21_0}}{s} + \sum_{i=1}^{m_{21}} \frac{2 k_{i_{21}} s}{s^2 + \omega_i^2} = \text{odd in } s = \frac{N_{21}}{D_{21}}$$

if N_{21} is odd, D_{21} is even

" is even " is odd

$$y_{22} = \frac{Ev\,D}{Od\,D} \text{ or } \frac{Od\,D}{Ev\,D}$$

$$\frac{V_2}{V_1} = \frac{N}{D} = \frac{N}{Ev\,D + Od\,D} = \left\{ \frac{N/Ev\,D}{1 + \frac{Od\,D}{Ev\,D}} \text{ or } \frac{N/Od\,D}{1 + \frac{Ev\,D}{Od\,D}} \right.$$

choice fixed by N odd or even

$$\frac{V_2}{V_1} = \frac{k}{s^3 + 2s^2 + 3s + 4} = \frac{N}{D}, \quad \underset{\text{even}}{N = k} = \text{a constant}$$

$$= \frac{k}{(s^3 + 3s) + (2s^2 + 4)} = \frac{k/(s^3+3s)}{1 + \left(\frac{2s^2+4}{s^3+3s}\right)}$$

$$\underset{(\text{odd})}{y_{21} = \frac{-k}{s(s^2+3)}}, \quad \underset{(\text{even/odd})}{y_{22} = \frac{2s^2+4}{s(s^2+3)}}$$

$$= \frac{-k/3}{s} + \frac{\frac{-k}{-3}s}{s^2+3} = -k\left\{\frac{\frac{1}{3}(s^2+3) - \frac{1}{3}s^2}{s(s^2+3)}\right. = \frac{-k}{s(s^2+3)}$$

Synthesize y_{22} to give y_{12}; all 3 zeros of y_{12} are at ∞

$\Rightarrow$ 1st Cauer synthesis

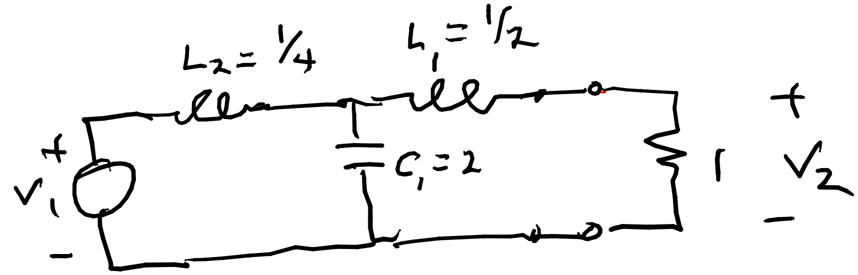
$y_{22} = \dfrac{2s^2+4}{s^3+3s}$ no pole at ∞, but one is in $\dfrac{1}{y_{22}} = \dfrac{s^3+3s}{2s^2+4}$

$z_{22} = \dfrac{1}{y_{22}} \Rightarrow$ $\Longrightarrow z$

$$\begin{array}{r|l} 2s^2+4 & s^3+3s \quad \to \tfrac{1}{2}s \\ & s^3+2s \\ & s \end{array}$$

$$\begin{array}{r|l} s & 2s^2+4 \quad \to 2s \leftarrow y \\ & 2s^2 \\ & 4 \end{array}$$

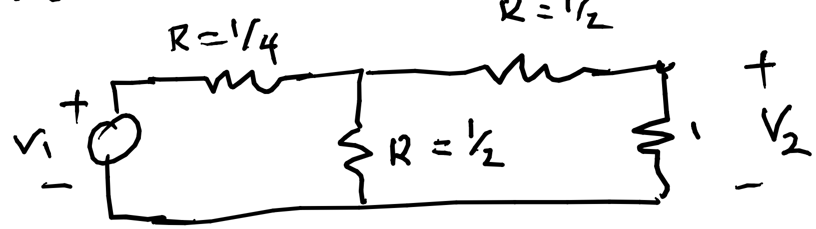
$$\begin{array}{r|l} 4 & s \quad \to \tfrac{1}{4}s \leftarrow z \end{array}$$



to find the k in y_{21}; let $s = 1$, $Ls = L\sigma = L$ looks like a resistor $R = L$

$Cs = C\sigma \Rightarrow G = C$

@ $\sigma = 1$ the circuit is



$$\frac{V_2}{V_1} = \frac{-y_{21}(1)}{1 + y_{22}(1)}$$

$$\frac{V_2}{V_1} = \frac{k}{s^3 + 2s^2 + 3s + 4}\Bigg|_{\sigma = 1} = \frac{k}{1+2+3+4} = \frac{k}{10}$$

here $k = 4$ as $\frac{V_2}{V_1} = \frac{1}{5/2} = \frac{k}{10}$

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to get state variables for a given transfer function

$$\frac{Y}{U}(s) = T(s) = \frac{M_1 s + M_2}{s^2 + d_1 s + d_0} \Rightarrow \begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array}$$

here $D = 0$

look at $\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -d_0 & -d_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u$

$$y = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

To get $T(s)$, $k=2$

$$\left.\begin{aligned} s1_k X &= AX + BU \\ Y &= CX + DU \end{aligned}\right\} \quad \begin{aligned} X &= (s1_k - A)^{-1} BU \\ Y &= \{C(s1_k - A)^{-1} B + D\} U \end{aligned}$$

$$T(s) = D + C(s1_k - A)^{-1} B \qquad D = T(\infty)$$

need $(s1_2 - A) = \begin{bmatrix} s & -1 \\ d_0 & s + d_1 \end{bmatrix}$; $\det(s1_2 - A) = s^2 + d_1 s + d_0$

$$(s1_2 - A)^{-1} = \frac{1}{s^2 + d_1 s + d_0} \begin{bmatrix} s + d_1 & +1 \\ -d_0 & s \end{bmatrix}$$

$$(s^2+d_1 s+d_0)\left[C(s\mathbb{1}_2-A)^{-1}B\right] = \begin{bmatrix} c_1 & c_2 \end{bmatrix}\begin{bmatrix} s+d_1 & 1 \\ -d_0 & s \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = n_1 s + n_0$$

$$= \begin{bmatrix} c_1 s + c_1 d_1 - c_2 d_0 , & c_1 + c_2 s \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

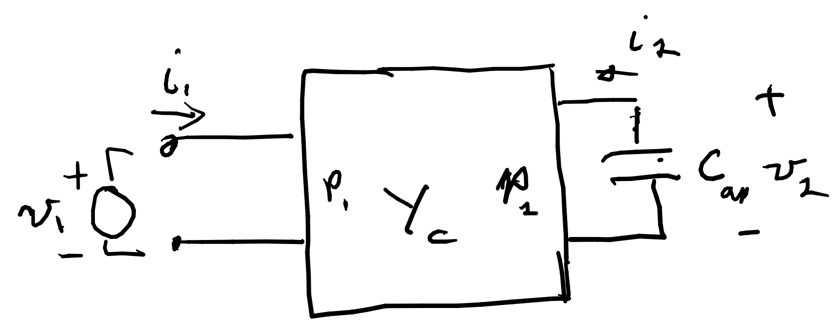
choose $b_1 = 0$, $b_2 = 1$

then

$$= c_1 + c_2 s = n_0 + n_1 s$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -d_0 & -d_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} n_0 & n_1 \end{bmatrix} x + d u$$



$$T(s) = \frac{i_1}{v_1}$$

$$i_2 = -C_{ap}\dot{v}_2 = -C_{ap}\dot{x}$$

$$\begin{matrix} y \\ -C_{ap}\dot{x} \end{matrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \overbrace{v_1}^{u} \\ v_2 \end{bmatrix} \begin{matrix} \\ \}\, x \end{matrix}$$

y_{11} is $p_1 \times p_1$

y_{22} is $p_2 \times p_2$

$p_1 = \#$ of input ports

$p_2 = \#$ of capacitors

$$-C_{ap}\dot{x} = y_{22}x + y_{21}u =$$

$$y = y_{12}x + y_{11}u = Cx + Du$$

$$\dot{x} = -C_{ap}^{-1}y_{22}x - C_{ap}^{-1}y_{21}u = Ax + Bu$$

$$Y_c = \begin{bmatrix} D & C \\ -C_{ap}B & -C_{ap}A \end{bmatrix}$$