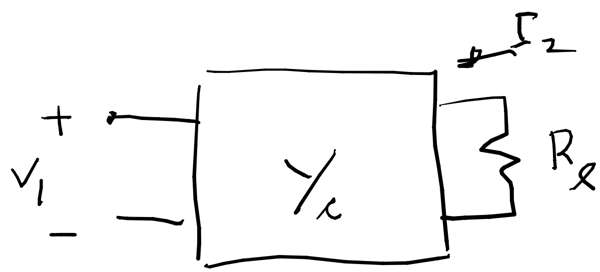
EE 610
10/12/10

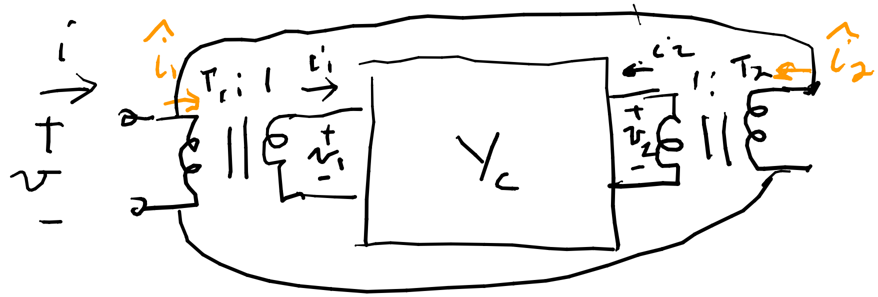


$V_2 = -R_\ell I_2 \Rightarrow I = -G_\ell V_2$

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2 = -G_\ell V_2$$

$$\frac{V_2}{V_1} = \frac{y_{21}}{-(G_\ell + y_{22})} = \frac{-y_{21}}{G_\ell + y_{22}} = \frac{-y_{21}/G_\ell}{1 + y_{22}/G_\ell}$$



$$i = y_{in} v \quad ; \quad v = T_1 v_1 = T_2 v_2 \Rightarrow v_1 = v/T_1, \quad v_2 = v/T_2$$

$$= \hat{i}_1 + \hat{i}_2 \qquad \hat{i}_1 = T_1 i_1, \quad \hat{i}_2 = T_2 i_2$$

$$= T_1 (y_{11} v_1 + y_{12} v_2) + T_2 (y_{21} v_1 + y_{22} v_2)$$

$$= \left\{ T_1 \left(\frac{y_{11}}{T_1} + \frac{y_{12}}{T_2} \right) + T_2 \left(\frac{y_{21}}{T_1} + \frac{y_{22}}{T_2} \right) \right\} v$$

$$= y_{in} v \Rightarrow y_{in} = y_{11} + y_{22} + \frac{T_1}{T_2} y_{12} + \frac{T_2}{T_1} y_{21}$$

if $y_{12} = y_{21}$ then $y_{in} = y_{11} + y_{22} + \left(\frac{T_1}{T_2} + \frac{T_2}{T_1} \right) y_{21}$

$$= y_{11} + y_{22} + \left(\frac{T_1^2 + T_2^2}{T_2 T_1} \right) y_{21}$$

choose $T_2 T_1 = \pm 1$ & look at a pole on $j\omega$ axis

y_{in} is PR if Y is PR

& $\left. y_{in} \right|_{\substack{\text{near} \\ \text{a pole} \\ \text{on } j\omega \text{ axis}}} \approx \dfrac{2k_1 s}{s^2+\omega_0^2} + \dfrac{2k_2 s}{s^2+\omega_0^2} + \dfrac{2k_{21} s}{s^2+\omega_0^2}$; $2k_1$ depends on $\dfrac{T_1^2+T_2^2}{T_1 T_2}$

if pole for y_{21} is not present in y_{11} & y_{22}

i.e. $k_1 = k_2 = 0$ $\left. y_{in} \right|_{\substack{\text{near} \\ s = j\omega_0}} \approx$ (any sign on k_{21} by choice of $T_1 T_2$)

$\Rightarrow$ can not be PR as can get a negative residue ($2k_{21}$) for a $j\omega$ axis pole.

If there is a common pole the residue

$$k_1 + k_2 + k_{21} \geq 0$$

$\therefore$ any pole on the $j\omega$ axis in y_{21} is also in y_{11} (and y_{22})

$$\frac{V_2}{V_1} = \frac{-y_{21}}{1 + y_{22}}$$

for a lossless coupling circuit then $y_{22}(s) = \dfrac{N_{22}(s)}{D_{22}(s)}$, $y_{21} = \dfrac{N_{21}}{D_{21}}$

$$= \frac{-N_{21} D_{22}}{D_{21}(D_{22} + N_{22})}$$

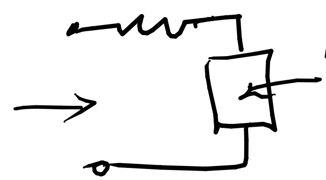
N & D polynomial

$$= \frac{-N_{21} \hat{D}_{22}}{\underbrace{D_{22} + N_{22}}_{\text{an even + an odd}}}$$

y_{22} = reactance function

So if given $\frac{V_2}{V_1} = \frac{N(s)}{D(s)}$; $D(s)$ has no zeros in $\sigma \geq 0$ as passive loaded in a resistor $=$ a Hurwitz polynomial

Look at



pr $\Rightarrow Z_{in} = R + \frac{N_{LC}}{D_{LC}} = \frac{1}{Y_{in}} \Rightarrow Y_{in} = \frac{D_{LC}}{RD_{LC} + N_{LC}} \Big|_{R=1} = \frac{D_{LC}}{D_{LC} + N_{LC}}$

has no pole in $\sigma \geq 0$ as pr & with loss

$\therefore$ the denominator + numerator of a reactance function is a Hurwitz polynomial

$\therefore$ synthesize y_{22} to get the zeros of y_{21} (easy if zeros of V_2/V_1 are at $s=0$ or $s=\infty$)

$$\frac{V_2}{V_1} = \frac{N}{D} \; ; \qquad y_{22} = \frac{\text{Od } D}{\text{Ev } D} \text{ or vice versa}$$

Ex: $$\frac{V_2}{V_1} = \frac{k}{s^2 + 10s + 5} \qquad D(s) = (s^2+5) + (10s)$$

$$\frac{\text{Od } D}{\text{Ev } D} = \frac{10s}{s^2+5}$$

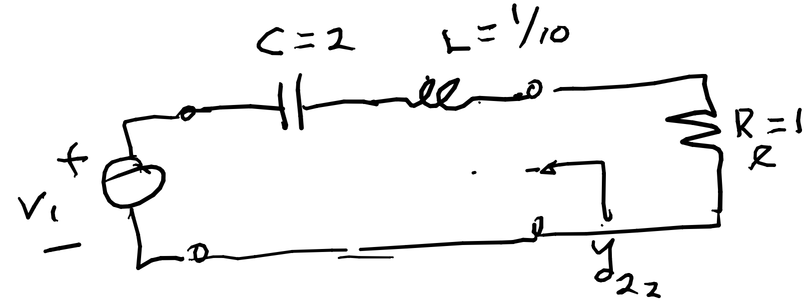
$$y_{22} = \frac{10s}{s^2+5} \, , \quad y_{21} = \frac{-k}{s^2+5} \; ;$$ k results from the synthesis

$y_{22} \rightarrow$ 

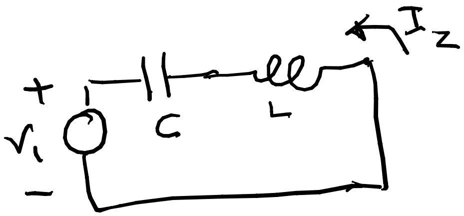
gives zeros of transmission at $s = \infty$ (where series arms open & shunt arms short)

$\Rightarrow$ 1st Cauer synthesis of y_{22}

$$\hat{z}_{22} = \frac{1}{y_{22}} = \frac{s^2+5}{10s} = \frac{1}{10}s + \frac{5}{10s} = \frac{1}{10}s + \frac{1}{\frac{10}{5}s}$$



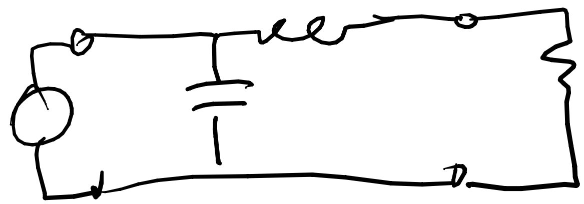
$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$



$$I_2 = -Y_1 V_1 = \frac{-1}{Ls+\frac{1}{Cs}} V_1$$

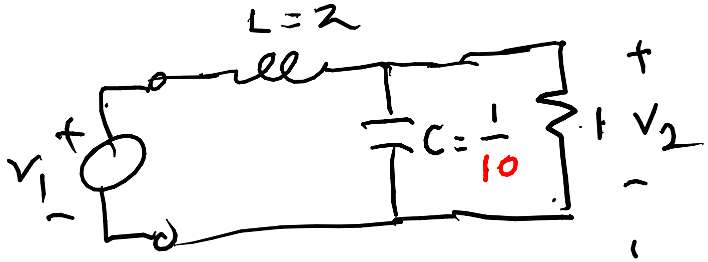
$$y_{21} = \frac{-Cs}{LCs^2+1} = \frac{-\frac{1}{L}s}{s^2+\frac{1}{LC}}$$

$$= \frac{ks}{s^2+5}$$



doesn't give y_{21}

look at $y_{22} = \frac{Ev}{Od} = \frac{s^2+5}{10s} = \frac{1}{10}s + \frac{1}{2s}$



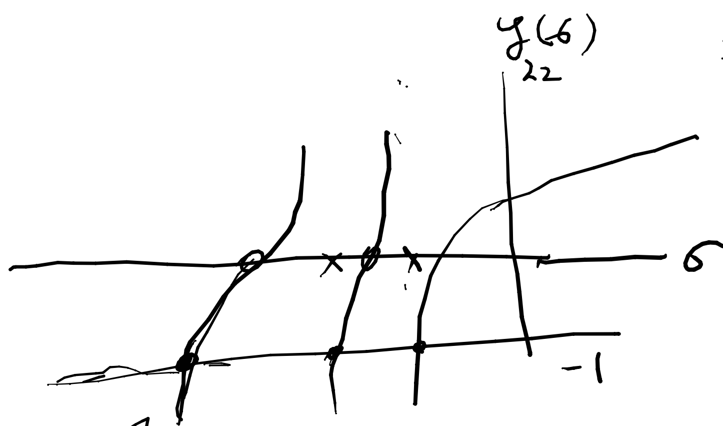
$y_{21} = -\frac{1}{2s}$

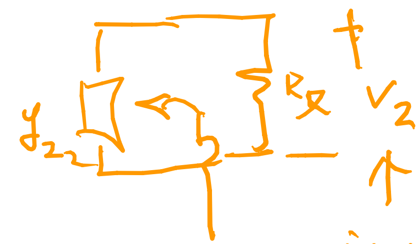
$$\frac{V_2}{V_1} = \frac{Z_{C,1}}{Z_L + Z_{C,1}} = \frac{\frac{1}{1+sC}}{sL + \frac{1}{1+sC}} = \frac{1}{s^2LC + sL + 1} = \frac{1/LC}{s^2 + \frac{s}{C} + \frac{1}{LC}}$$

$$\frac{V_2}{V_1} = \frac{5}{s^2 + 10s + 5} = \frac{k}{s^2+10s+5}, \quad k = 5$$

For RC synthesis $\frac{V_2}{V_1} = \frac{-y_{21}}{\hat{y}_{22}}$ all poles are on the $-\sigma$ axis

$= \frac{-y_{21}}{1+y_{22}}$ if terminated in $R_\ell = 1$



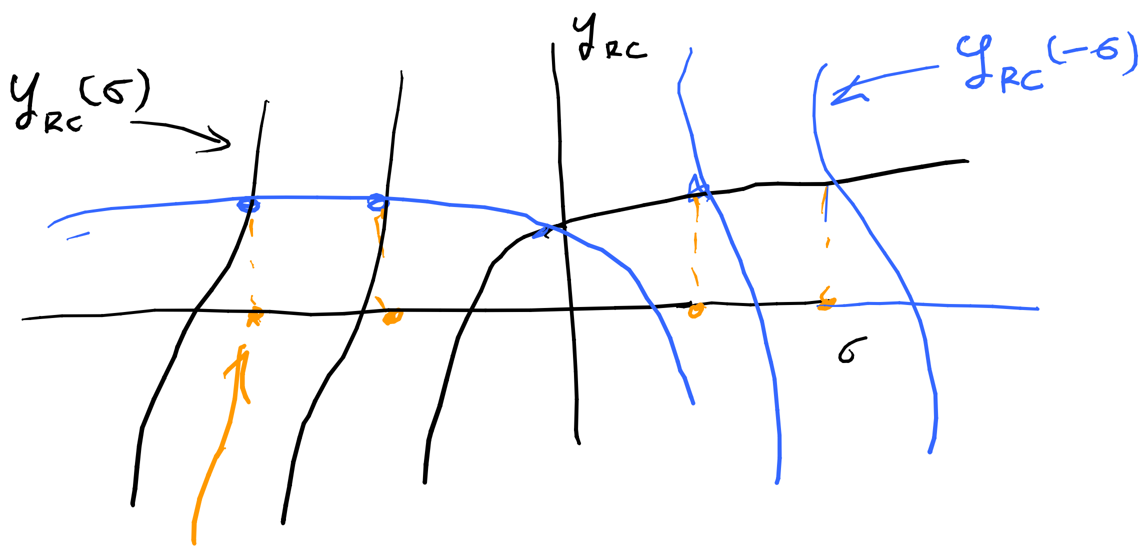


is on parallel combination of R_ℓ & y_{22}

intersects are poles for $\frac{V_2}{V_1}$

Zeros of the even part of an RC circuit are all real

$y_{RC} + y_{RC_*}$



zeros of the even part of y_{RC}

allows to use real k in Richards' functions for $y_{RC}(x)$