EE 610
10/07/10

RC driving point syntheses from LC reactance function

lossless pr $y(s) \Rightarrow y(s) + y(-s) = y + y_* = 0$

$Ev\, y \equiv 0$

$\Downarrow$

all poles on $j\omega$ axis & simple with positive residues

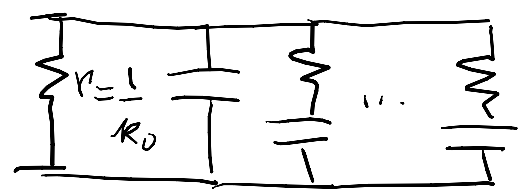
$$y_{LC}(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n} \frac{2k_i s}{s^2 + \omega_{0i}^2} \qquad k_i = \text{residues} > 0$$

$$= s \frac{\prod(s^2 + \omega_{0i}'^2)}{\prod(s^2 + \omega_{0i}^2)} \leftarrow \text{simple zeros } z = 1/y \text{ is pr}$$

Even $= F(s^2)$

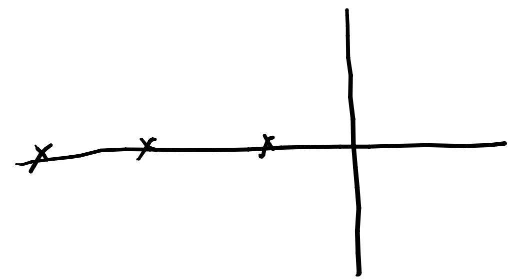
$$= s F(s^2)$$

$$y_{RC} = k_0 + k_\infty s + \sum \frac{2k_i s}{s + \omega_{0i}^2}$$



$$\frac{y_{RC}}{s} = \frac{k_0}{s} + k_\infty + \sum_{i=1}^{n} \frac{2k_i}{\underbrace{s + \omega_{0_i}^2}_{>0}} \Rightarrow \text{partial fraction expansion of } y/s$$

$k_i > 0$, $\omega_{0_i}^2 > 0$

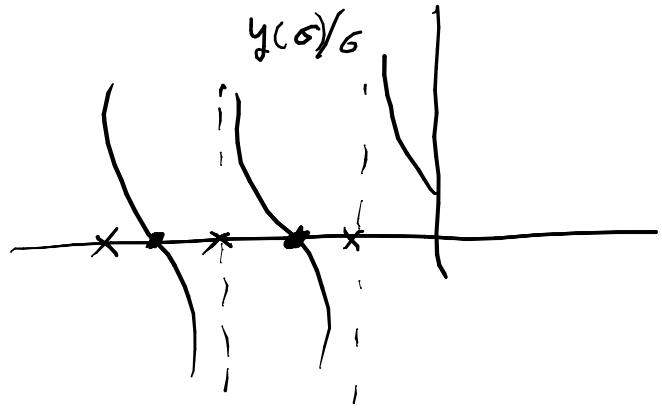


for synthesis divide y_{RC}/s and get the partial fraction expansion. There is no pole at $s=0$, can be one at ∞

set $s = \sigma$ real

$$\left.\frac{d\, y_{RC}/s}{ds}\right|_{s=\sigma} = \frac{d\left(\frac{k_0}{\sigma} + k_\infty + \sum \frac{2k_i}{\sigma+\sigma_i^2}\right)}{d\sigma} = -\frac{k_0}{\sigma^2} + 0 + \sum \frac{2k_i(-1)}{(\sigma+\sigma_i^2)^2}$$

≤ 0 for a pr RC $y(s)$



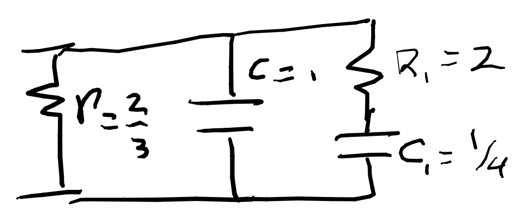
← shows poles & zeros alternate on $-\sigma$ axis

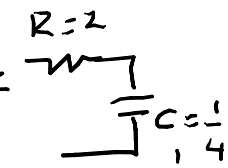
$y_{RC} - k_0 \Rightarrow$ still pr & $\dfrac{1}{y_{RC} - k_0}$ has a pole at ∞

Ex : $y(s) = \dfrac{(s+1)(s+3)}{(s+2)}$

$\dfrac{y}{s} = \dfrac{(s+1)(s+3)}{s(s+2)} = \dfrac{3/2}{s} + \dfrac{\frac{(-1)(+1)}{(-2)} = \frac{1}{2}}{s+2} + \left(\dfrac{s^2}{s^2} = 1\right)$

$y(s) = \dfrac{3}{2} + s + \dfrac{(1/2)\,s}{s+2} \Rightarrow$



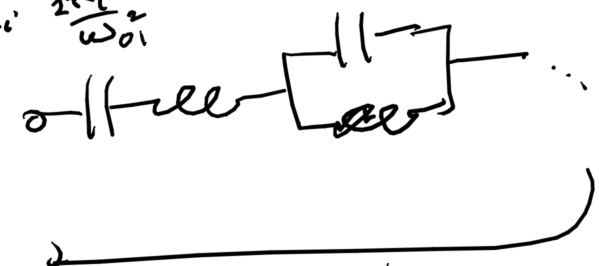
$y_1 = \dfrac{\frac{1}{2}s}{s+2}$; $z_1 = \dfrac{1}{y_1} = \dfrac{s+2}{\frac{1}{2}s} = 2 + \dfrac{1}{s/4} =$ 

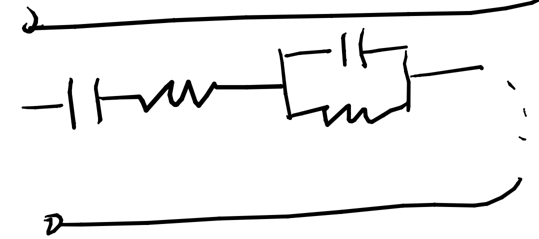
$z(s) = \dfrac{s+2}{(s+1)(s+3)}$

$z_i = \dfrac{1}{\frac{s}{2k_i} + \frac{1}{\frac{2k_i'}{\omega_{0i}^2}s}} \leftarrow$ inductor term

$\parallel$

return to LC case

$z = \dfrac{k_0'}{s} + k_\infty' s + \sum \dfrac{2k_i' s}{s^2 + \omega_{0i}^2} =$ 

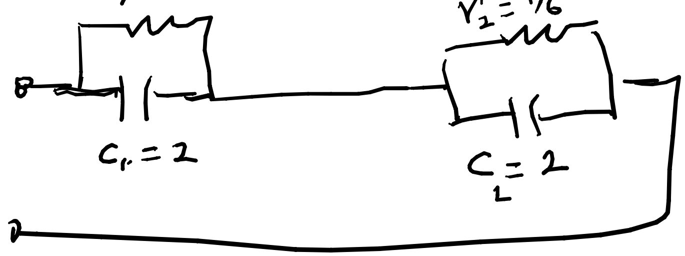
$\Rightarrow$ 

$= \dfrac{k_0'}{s} + k_\infty' + \sum \dfrac{1}{\frac{s}{2k_i} + \frac{\omega_{i0}^2}{2k_i}}$

$= \dfrac{k_0'}{s} + k_\infty' + \sum \dfrac{2k_i}{s + \omega_{0i}^2} \leftarrow$ residue at the pole at $s = -\omega_{0i}^2$

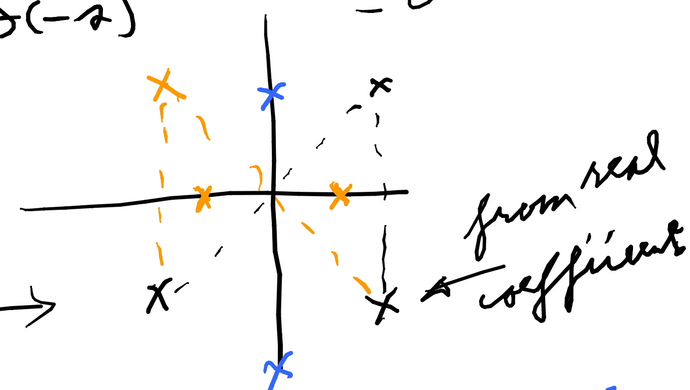
$$\therefore\ z = \frac{s+2}{(s+1)(s+3)} = \frac{\frac{-1+2}{-1+3}}{s+1} + \frac{\frac{-3+2}{-3+1}}{s+3} = \frac{1/2}{s+1} + \frac{1/2}{s+3}$$

$$= \frac{\frac{1}{2}(s+3) + \frac{1}{2}(s+1)}{(s+1)(s+3)} = \frac{s+2}{(s+1)(s+3)}$$



Even part zeros $2\,\mathrm{Ev}\,f(s) = f(s) + f(-s) = 0$
of rational $f(s)$ (scalar) $= F(s^2)$
with real coefficients

pole of $f(-s)$ if a pole of $f(s)$ →



holds for zeros & poles

For simple real zeros $(s+a)(-s+a) = -s^2+a^2$

$s^2 = x \Rightarrow -x+a^2$

if not simple but real $(s+a)^k(-s+a)^k = (-x+a^2)^k$

Real problem is to factor out complex zeros of polynomials with real coefficients

$$p(x) = a_p x^p + a_{p-1} x^{p-1} + \dots + a_1 x + a_0$$

$$= a_0\left(\frac{a_p}{a_0} x^p + \dots + \frac{a_1}{a_0} x + 1\right)$$

$$\hat{x}^p = \left(\frac{a_p}{a_0}\right) x^p$$

$$\hat{P}(\hat{x}) = \frac{P(x)}{a_0} = \hat{x}^p + \hat{a}_{p-1}\hat{x}^{p-1} + \dots + \hat{a}_1 + 1$$

$$\hat{x} = \sqrt{\frac{a_p}{a_0}}\, x$$

$$p(x) = (x^2 + a x + 1)\left(\frac{a_p}{a_0} x^{p-2} + \dots + 1\right)$$

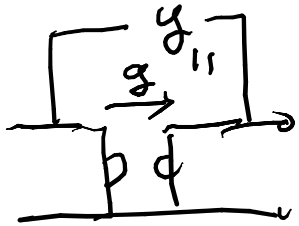
$$
\begin{array}{r|l}
 & \alpha_{p-2}x^{p-4} + \cdots + \alpha_2 \\
\hline
x^2 + ax + 1 & \alpha_{p-2}x^{p-2} + \cdots + \alpha_1 x + 1 \\
 & \qquad + \; b_1 x + b_0 \\
\hline
 & (\alpha_1 - b_1)x + (1 - b_0)
\end{array}
$$

$$(\underbrace{\alpha_1 - b_1}_{= \text{rem}_1})x + (\underbrace{1 - b_0}_{= \text{rem}_2})$$

desire to force $\underbrace{\alpha_1 - b_1}_{\text{rem}_1}$ & $\underbrace{1 - b_0}_{\text{rem}_2}$ to zero

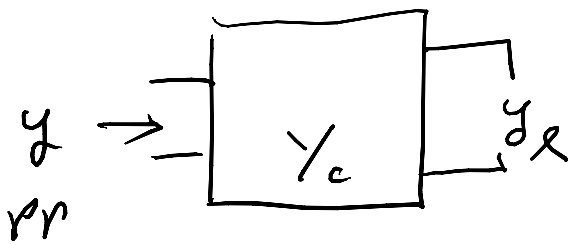
replace $x^2 + ax + 1 \Rightarrow x^2 + (a + \Delta_a)x + (1 + \Delta_1)$
so then repeat & by iteration obtains the degree 2 factor.

For the Richards' function it use real to



$\Rightarrow \quad Y_c = \begin{bmatrix} y_{11} & -y_{11}+g \\ -y_{11}-g & y_{11} \end{bmatrix}$; $\det Y = g^2$

$y_{11} = pr$



$$y = y_{11} - y_{12}\frac{1}{(y_{22}+y_\ell)}y_{21} = \frac{\Delta y + y_{11}y_\ell}{y_{22}+y_\ell}$$

here $$= \frac{g^2 + y_{11}y_\ell}{y_{11}+y_\ell}$$

$$S_y = \frac{y+g}{g-y} = \frac{\dfrac{g^2+y_{11}y_\ell}{y_{11}+y_\ell}+g}{\dfrac{g^2+y_{11}y_\ell}{y_{11}+y_\ell}-g} = \frac{g^2+y_{11}y_\ell+gy_{11}+gy_\ell}{g^2+y_{11}y_\ell-gy_{11}-gy_\ell}$$

$$= \frac{g(g+y_\ell)+y_{11}(g+y_\ell)}{g(g-y_\ell)-y_{11}(g-y_\ell)}$$

$$= \frac{(g+y_{11})(g+y_{\ell})}{(g-y_{11})(g-y_{\ell})} = \left(\frac{g+y_{11}}{g-y_{11}}\right)\left(\frac{g+y_{\ell}}{g-y_{\ell}}\right)$$

if S_y is br this shows S_{y_ℓ} is br & hence y_ℓ is pr

$\Rightarrow$ Richards' function is pr as $y_{11} = sc$ then