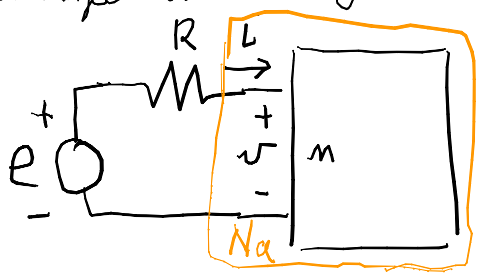
EE 610

10/05/10

Compilation of results



n-port or n terminal pairs

If linear

$A(s)\,v = (B(s)G)\,Ri$

$R = r\,1_n, \quad G = R^{-1} = g\,1_n, \quad g = 1/r$

$$\left.\begin{aligned} e = 2v^i &= v + Ri \\ 2v^r &= v - Ri \end{aligned}\right\} \quad \begin{aligned} v &= v^i + v^r \\ Ri &= v^i - v^r \end{aligned} \quad \Rightarrow \quad (BG - A)\,v^i = (BG + A)\,v^r$$

$$\begin{aligned} Ri &= RY_a e \\ v &= (1_n - RY_a)e \end{aligned} \quad \Rightarrow \quad \left.\begin{aligned} RY_a v &= RY_a e - RY_a RY_a e \\ &= Ri - RY_a(Ri) \end{aligned}\right\} \quad \begin{aligned} A &= RY_a \\ BG &= 1_n - RY_a \end{aligned}$$

$Zi = v, \quad Yv = i$

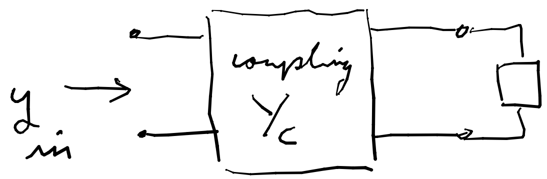
$$\Rightarrow \quad (1_n - 2RY_a)\,v^i = 1_n v^r$$

$$v^r = S v^i \Rightarrow \quad S = (BG - A)^{-1}(BG + A) = 1_n - 2RY_a$$

$$= (1_n - RY)^{-1}(1_n + RY) = (GZ - 1_n)^{-1}(GZ + 1_n)$$

$$Y = G(1_n + S)^{-1}(1_n - S) = Z^{-1}, \quad Z = (1_n - S)^{-1}(1_n + S)R = Y^{-1}$$

here $(1_n+X)^{-1}(1_n-X) = (1_n-X)(1_n+X)^{-1}$ $\left((1_n+X)^{-1} \,\&\, (1_n-X) \text{ commute}\right)$

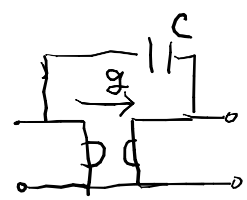


y_ℓ, $\quad Y_c = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$; $y_{in} = y_{11} - y_{12}(y_\ell + y_{22})^{-1} y_{21}$

For 2-port Y_c; $\quad y_{in} = \dfrac{\Delta y + y_\ell y_{11}}{y_{22} + y_\ell}$; Δy = determinant Y_c
$= y_{11} y_{22} - y_{12} y_{21}$

$$y_\ell = \frac{\Delta y - y_{22} y_{in}}{y_{in} - y_{11}}$$

For



$$Y_c = \begin{bmatrix} sc & -sc+g \\ -sc-g & sc \end{bmatrix}$$

$$y_\ell = \frac{g^2 - sC\, y_{in}}{y_{in} - sc} = \frac{(g^2/c) - s\, y_{in}(s)}{(1/c)\, y_{in}(s) - s}$$

$$= y(k)\left[\frac{k\, y(k) - s\, y(s)}{k\, y(s) - s\, y(k)}\right]$$

$g^2/c = k\, y(k)$

$\dfrac{k}{y(k)} = 1/c$

$g^2 = y(k)^2$

identify with the Richards' function $\times\, y(k)$

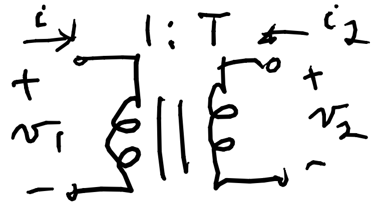
For a passive n-port $Y(s)$ & $Z(s)$ are positive-real when they exist while $S(s)$ (almost) always exists and is bounded real. For finite circuits these are rational (= ratio of 2 polynomials $\Rightarrow$ all singularities are poles) with real coefficients, i.e. PR for Y & Z and BR for S [write pr or br for scalars ($n=1$)]. And if $y(s)$ is pr so is the Richards' function.

$S(s)$ has no poles on $s=j\omega$ while Z & Y have at most simple poles on $s=j\omega$. Use lower $*$ for $s \to -s$. Then for lossless circuits, $0_n = n\times n$ zero matrix

$$Y+Y_* = 0_n \,, \quad Z+Z_* = 0_n \,, \quad 1_n = S_* S \Rightarrow S^{-1} = S_*$$

Transformer, gyrator equivalents, lossless synthesis, RC synthesis

2-port transformer



$v_2 = T v_1$

$P_{in}(t) \equiv 0$

$P_{in} = v_1 i_1 + v_2 i_2 = 0$

$= v_1 i_1 + T v_1 i_2 = v_1 (i_1 + T i_2) = 0$ for any v_1

$\Rightarrow i_1 = -T i_2$

$Av = Bi$

$$\begin{bmatrix} 0 & 0 \\ -T & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$Z = A^{-1}B$, $Y = B^{-1}A$ but no inverse for A or B

assume $G = 1_2$

$$S = (B+A)^{-1}(B-A)$$

$$= \begin{bmatrix} 1 & T \\ -T & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & T \\ T & -1 \end{bmatrix} = \frac{1}{1+T^2} \begin{bmatrix} 1 & -T \\ T & 1 \end{bmatrix} \begin{bmatrix} 1 & T \\ T & -1 \end{bmatrix}$$

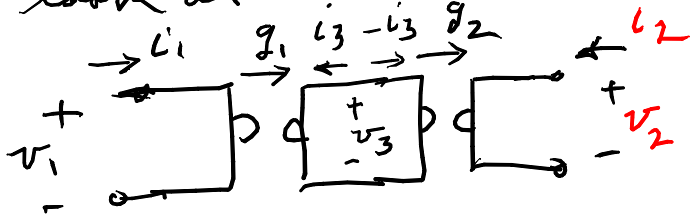
$$= \frac{1}{1+T^2} \begin{bmatrix} 1-T^2 & 2T \\ 2T & -1+T^2 \end{bmatrix}$$

$$1_2 - S^T S \Rightarrow S^2 = S^T S = \frac{1}{(1+T^2)^2} \begin{bmatrix} 1-T^2 & 2T \\ 2T & -1+T^2 \end{bmatrix} \begin{bmatrix} 1-T^2 & 2T \\ 2T & -1+T^2 \end{bmatrix}$$

$$= \frac{1}{(1+T^2)^2} \begin{bmatrix} (1-T^2)^2 + 4T^2 & 0 \\ 0 & 4T^2 + (-1+T^2)^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ checks lossless condition

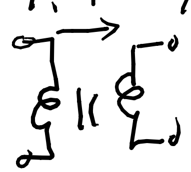
look at



$i_1 = g_1 v_3$ $\quad -i_3 = g_2 v_2$

$i_3 = -g_1 v_1$ $\quad i_2 = -g_2 v_3$

$1 : T = g_1/g_2$

$\equiv$ 

$$i_1 = g_1\left(\frac{-i_2}{g_2}\right) \Rightarrow i_1 = -\frac{g_1}{g_2} i_2 \Rightarrow i_1 + \frac{g_1}{g_2} i_2$$

$$v_2 = -\frac{i_3}{g_2} = +\frac{g_1}{g_2} v_1 \Rightarrow v_2 = \frac{g_1}{g_2} v_1$$

Lossless synthesis $n=1$

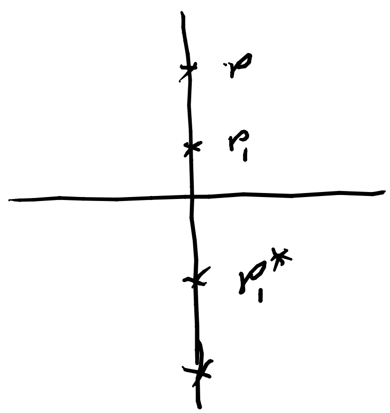
$$y(s) + y(-s) = y + y_* = 2\,\mathrm{Ev}\,y$$

$$y - y_* = 2\,\mathrm{Od}\,y$$

$$y = \mathrm{Ev}\,y + \mathrm{Od}\,y$$

$$\mathrm{Ev}\,y \equiv 0$$

$$y(s) = s\left(\frac{\prod s^2+\omega_i^2}{\prod s^2+\omega_i'^2}\right)$$



$$y = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n_1} \frac{2k_i s}{s^2+\omega_i^2} \qquad \text{pr}$$

$$z = \frac{k_0'}{s} + k_\infty' s + \sum_{i=1}^{n_1'} \frac{2k_i' s}{s^2+\omega_i'^2} \qquad \text{also pr}$$

y no pole at $s=0$ in y there is one in z

$$\frac{1}{\left(y - \frac{k_0}{s}\right)} = \frac{k_0'}{s} + \cdots$$

same at $s = \infty$

Ex:

$$y = \frac{(s^2+1)(s^2+5)}{s(s^2+4)} \quad \text{has a pole at } s = \infty$$

$$= \frac{s^4 + 6s^2 + 5}{s^3 + 4s}$$

$$\begin{array}{r|l} s^3 + 4s & s^4 + 6s^2 + 5 \quad (s) \\ & s^4 + 4s^2 \\ \hline & 2s^2 + 5 \end{array}$$

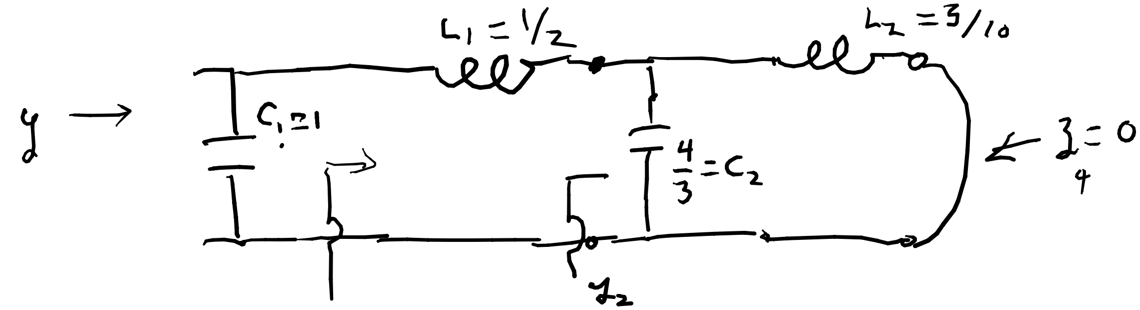
$$\begin{array}{r|l} 2s^2 + 5 & s^3 + 4s \quad (\tfrac{1}{2}s) \\ & s^3 + \tfrac{5}{2}s \\ \hline & \tfrac{3}{2}s \end{array}$$

$$\begin{array}{r|l} \tfrac{3}{2}s & 2s^2 + 5 \quad (\tfrac{4}{3}s) \\ & 2s^2 \\ \hline & 5 \end{array}$$

$$\begin{array}{r|l} 5 & \tfrac{3}{2}s \quad (\tfrac{3}{10}s) \\ & \tfrac{3}{2}s \\ \hline & 0 \end{array}$$

$$y = s + \cfrac{1}{\tfrac{1}{2}s + \cfrac{1}{\tfrac{4}{3}s + \cfrac{1}{\tfrac{3}{10}s + 0}}}$$

= continued fraction expansion about $s = \infty$



= 1st Cauer synthesis removed poles at $s = \infty$

$$z_1 = \frac{1}{2}s + \cfrac{1}{\underbrace{\frac{4}{3}s + \cfrac{1}{\frac{3}{10}s}}}$$

$$y_2 = \frac{4}{3}s + \cfrac{1}{\frac{3}{10}s + \underset{\substack{\uparrow \\ z_4}}{0}}$$

gives zeros of transmission at $s = \infty$
$\Rightarrow$ low pass filter

2nd Cauer = remove poles at $s = 0$

$$y = \frac{s^4 + 6s^2 + 5}{s^3 + 4s} = \frac{5 + 6s^2 + s^4}{4s + s^3}$$

$$4s + s^3 \,\big)\, 5 + 6s^2 + s^4 \quad \frac{5}{4s}$$

$$5 + \frac{5}{4}s^2$$

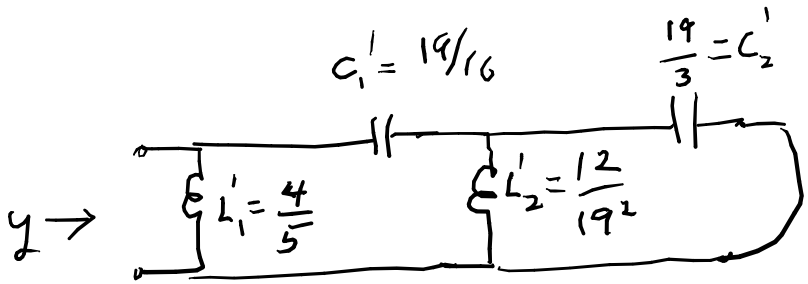
$$\frac{19}{4}s^2 + s^4 \,\big)\, 4s + s^3 \quad \frac{16}{19s}$$

$$4s + \frac{16}{19}s^3$$

$$\frac{3}{19}s^3 \,\big)\, \frac{19}{4}s^2 + s^4 \quad \frac{19^2}{12s}$$

$$\frac{19}{4}s^2$$

$$s^4 \,\big)\, \frac{3}{19}s^3 \quad \frac{3}{19s}$$



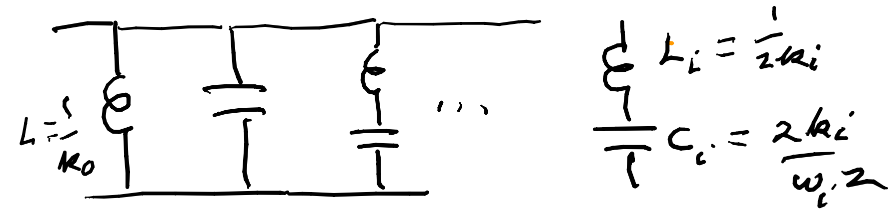
2nd Cauer

(gives zeros of transmission at $s = 0$ if attach output leads) $\Rightarrow$ high pass filter

see chapter 8

LC $\quad y_{LC}(s) = \frac{k_0}{s} + k_\infty s + \left(\sum_{i=1}^{m_1} \frac{2k_i}{s + \frac{\omega_i^2}{s}} = \sum \frac{1}{\frac{s}{2k_i} + \frac{\omega_i^2}{2k_i s}} \right)$

LC



$\Downarrow$

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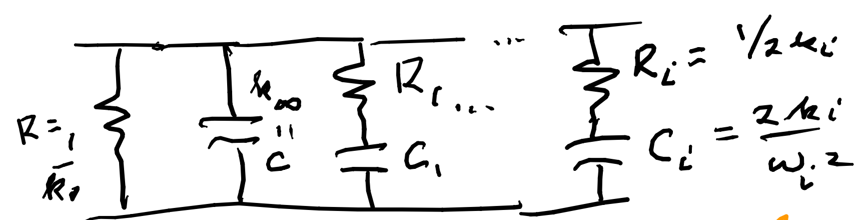
RC



$$y_{RC} = k_0 + k_\infty s + \left(\sum_{i=1}^{m_1} \frac{1}{\frac{1}{2k_i} + \frac{\omega_i^2}{2k_i s}} = \sum_{i=1}^{m_1} \frac{2k_i s}{s + \omega_i^2} \right)$$