EE 610
09/30/10

Main passive condition

$$\mathrm{Re}\, V^{T*}(s)\, I(s) \geq 0$$

$$\frac{V^{T*} I + I^{T*} V}{2} = \frac{V^{T*}(Y)V + V^{T*} Y^{T*} V}{2}$$

$$= V^{T*}\left(\frac{Y(s) + Y^{T*}(s)}{2}\right) V \geq 0$$

$$\frac{Y(s) + Y^{T*}(s)}{2} = \text{Hermitian part of } Y$$

$r = z_0$ $\quad R = r\,1_n$

$G = \frac{1}{r} 1_n$

$$V^{T*} I = (V^i + V^r)^{*T} G(RI)$$

$$= \left(\frac{V^{iT*} + V^{rT*}}{2}\right) G \left(\frac{V^i - V^r}{2}\right)$$

$$4V^{T*} I = V^{iT*} G V^i - V^{iT*} G V^r + V^{rT*} G V^i - V^{rT*} G V^r$$

$$0 \leq v^{iT*} G v^{i} - \underset{v^{iT*}S^{T*}}{\underset{\shortparallel}{v^{rT*}}} G \underset{Sv^{i}}{\underset{\shortparallel}{v^{r}}}$$

$$= v^{iT*}\left[G - S^{T*} G S\right] v^{i} \Rightarrow \underbrace{1_n - S^{T*}(s) S(s)}_{\text{Hermitian form is } \geq 0 \text{ in } \sigma \geq 0}$$

Bounded real condition

for $s = j\omega \Rightarrow 1_n - S^{T}(-j\omega) S(j\omega) = 0_n$ if lossless

extend by $\omega = s/j$. $\Rightarrow 1_n - S^{T}(-s) S(s) = 0_n$

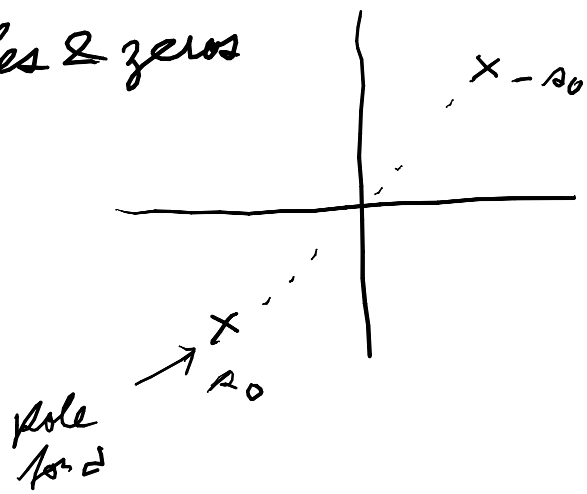
$\Rightarrow 1_n - S_*^{T} S = 0_n$ for all s if rational & lossless

$$Y^{T}(-s) + Y(s) = 0_n$$

$n = 1$, scalar $y(s)$: if PR & lossless

$$y(s) = -y(-s) = \frac{n(s)}{d(s)} = -\frac{n(-s)}{d(-s)}$$

Poles & zeros



since on RH side also on left so can't be

$\therefore$ no poles in $\sigma < 0$ for LPR $=$ lossless

$$y + y_* = \frac{1}{z} + \frac{1}{z_*} = \frac{z + z_*}{z z_*} \Rightarrow \text{a zero of } y \to \text{pole of } z$$

$$\therefore z + z_* = 0$$

$\therefore$ an LPR function has only poles & zeros on the $j\omega$ axis

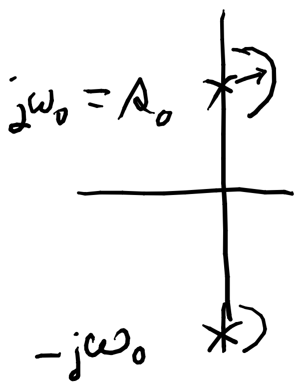
$2\,\mathrm{Ev}\,y = y + y_*$ $\quad\quad$ $y = \mathrm{Ev}\,y + \mathrm{Od}\,y$

$2\,\mathrm{Od}\,y = y - y_*$ $\quad\quad$ if lossless $\mathrm{Ev}\,y = 0$

an LPR $y(s)$ is an odd function of s

Ex: $\quad y(s) = \dfrac{2s}{3s^2+5}\,, \quad y(-s) = \dfrac{-2s}{3s^2+5} \Rightarrow y + y_* = 0$

$y_2(s) = \dfrac{2s(3s^2+8)}{(3s^2+5)}$ also odd



check $\mathrm{Re}\, Y(s)$ in $\sigma > 0$

$\Rightarrow$ pole is simple with a positive

$\approx$ near the pole $\quad Y(s) \approx \dfrac{k}{s - j\omega_0}$, $k \geq 0$

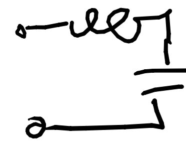
also $\quad + \dfrac{k_2}{s + j\omega_0}$

$$Y(s) \approx \frac{k(s + j\omega_0) + k_2(s - j\omega_0)}{(s + j\omega_0)(s - j\omega_0)}$$

real coefficients requires $k_2 = k$

$$= \frac{2ks}{s^2 + \omega_0^2}$$

$$\Rightarrow Y(s) = \frac{2ks}{s^2 + \omega_0^2} \Rightarrow Z = \frac{1}{Y} = \frac{s^2 + \omega_0^2}{2ks} = \frac{s}{2k} + \frac{1}{\frac{2k}{\omega_0^2} s}$$



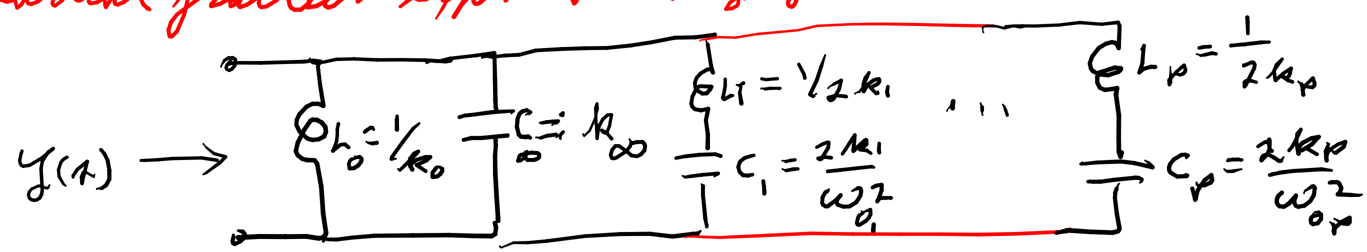
$L = \frac{1}{2k}$

$C = 2k/\omega_0^2$

given an LPR $Y(s)$

$$Y(s) = \sum_{i=1}^{p} \frac{2k_i s}{s^2+\omega_{0_i}^2} + \frac{k_0}{s} + k_\infty s \quad , \quad k_i \geq 0$$

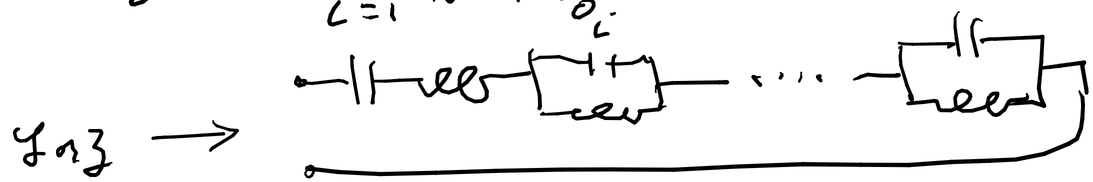
partial fraction expansion of $Y(s)$



= 2nd Foster synthesis of an LPR immitance

can do the same with Z

$$Z(s) = \sum_{i=1}^{p'} \frac{2k_i' s}{s^2+\omega_{0_i}'^2} + \frac{k_0'}{s} + k_\infty' s$$



1st Foster

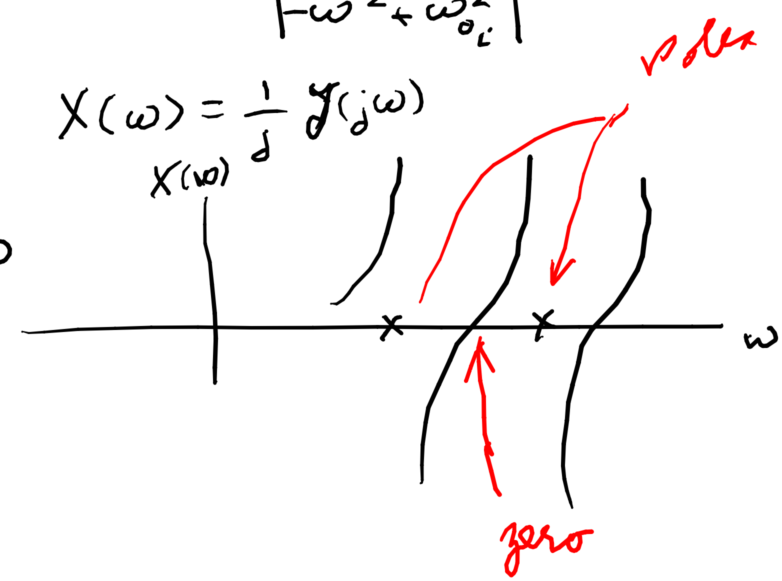
$$\frac{d\, y(j\omega)}{d\omega} = \frac{d}{d\omega}\left(\frac{k_0}{j\omega} + k_\infty j\omega + \sum_{i=1}^{p} \frac{2k_i\, j\omega}{-\omega^2+\omega_{0_i}^2}\right)$$

$$\left(\frac{1}{j}\right)\frac{dy}{d\omega} = +\frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{p}\left(\frac{2k_i}{-\omega^2+\omega_{0_i}^2} - \frac{(2k_i\omega)(-2\omega)}{(-\omega^2+\omega_{0_i}^2)^2}\right)$$

$$= \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{p}\left(\frac{2k_i\omega^2 + 2k_i\omega_{0_i}^2}{\left|-\omega^2+\omega_{0_i}^2\right|^2}\right) > 0 \text{ for all } \omega$$

$$y(j\omega) = jX(\omega) \Rightarrow X(\omega) = \frac{1}{j}\, y(j\omega)$$

$$\frac{d\,X(\omega)}{d\omega} \geq 0$$



Ex: $y(s) = \dfrac{1\,(s^2+1)(s^2+5)}{s\,(s^2+4)} = \dfrac{k_0}{s} + \dfrac{k_1}{s+j2} + \dfrac{k_2}{s-j2} + k_\infty s$

$k_2 = k_1$

$k_0 = s\,y(s)\Big|_{s=0} = \dfrac{\cancel{s}}{\cancel{s}}\dfrac{(s^2+1)(s^2+5)}{s^2+4}\Big|_{s=0} = \dfrac{s\,k_0}{s} + \underbrace{\dfrac{k_1 s}{s+j2} + \dfrac{k_2 s}{s-j2} + k_\infty s^2}_{0}\Big|_{s=0}$

$= 5/4 \qquad\qquad = k_0$

$\dfrac{2k_1 s}{s^2+4} \Rightarrow$ to get k_1; $\dfrac{s}{s^2+4}\cdot\dfrac{1}{s} \times y(s)$ set $s^2 = -4$

$2k_1 = \dfrac{\cancel{s^2+4}}{s} \times \dfrac{1}{s}\left(\dfrac{(s^2+1)(s^2+5)}{\cancel{s^2+4}}\right)\Big|_{s^2=-4} = \dfrac{1}{-4}\,(-4+1)(-4+5) = \dfrac{3}{4} \times 1$

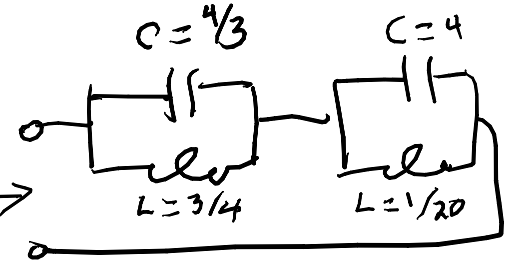
$y(s) = \dfrac{5/4}{s} + 1\cdot s + \dfrac{3/4\, s}{s^2+4}$ = partial fraction expansion

$$z(s) = \frac{1}{y(s)} = \frac{s(s^2+4)}{(s^2+1)(s^2+5)} = \frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+5} = \frac{\frac{3}{4}s}{s^2+1} + \frac{\frac{1}{4}s}{s^2+5}$$

$$2k_1 = \left.\frac{s^2+4}{s^2+5}\right|_{s^2=-1} = \frac{3}{4}, \quad 2k_2 = \left.\frac{s^2+4}{s^2+1}\right|_{s^2=-5} = \frac{-1}{-4} = \frac{1}{4}$$

1st Foster, z $\rightarrow$

$$y = \frac{(s^2+1)(s^2+5)}{s(s^2+4)} \rightarrow$$



2nd Foster, y

$$\frac{\frac{3}{4}s}{s^2+4} = y$$

$$y = \frac{(s^2+1)(s^2+5)}{s(s^2+4)} \rightarrow$$