09/28/10

EE610

modified after class

$$2v^i = v + Ri$$
$$2v^r = v - Ri$$

$$R = r\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & \cdots & \cdots & 1 \end{bmatrix} = r1_m = G^{-1}$$

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

= S for circulator

$$V^r = SV^i$$
$$= \frac{V - RI}{2} = S\frac{(V + RI)}{2}, \quad S(s)$$
$$s = \sigma + j\omega$$

to find Y from S

$$V - SV = RI + SRI$$
$$(1_m - S)V = (1_m + S)RI = (1_m + S)RYV$$
$$(1_m + S)^{-1}(1_m - S)V = RYV$$
$$Y = R^{-1}(1_m + S)^{-1}(1_m - S) = R\left[(1_m - S)(1_m + S)^{-1}\right]$$

i.e.: we can exchange order of $(1_m + S)^{-1}$ and $(1_m - S)$

Since $\quad (1_n+X)^{-1}(1_n-X) \stackrel{?}{=} (1_n-X)(1_n+X)^{-1}$

$\times(1_n+X)$ on right & left $\quad (1_n-X)(1_n+X) = (1_n+X)(1_n-X)$

$$= 1_n+\cancel{X}-\cancel{X}-X^2 \qquad = 1_n-\cancel{X}+\cancel{X}-X^2$$

$$\det(1_3+S) = \det\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = 1\times 1 - (1)\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 2$$

$$(1_3+S)^{-1} = \frac{1}{2}\begin{bmatrix} 1 & +1 & -1 \\ -1 & 1 & +1 \\ +1 & -1 & 1 \end{bmatrix}$$

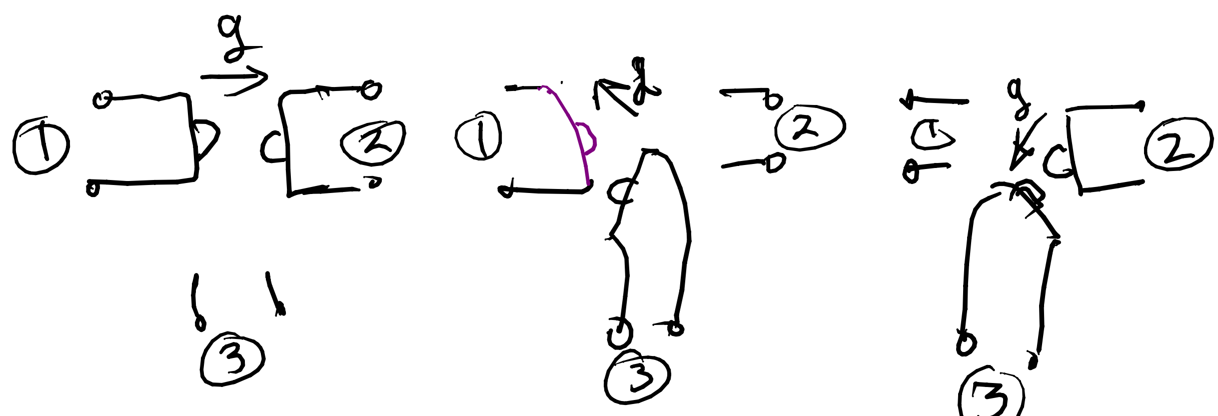
$$\Delta_{21} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1$$

$$(1,2)\text{ entry} = (-1)^{2+1}\times\Delta_{21}$$

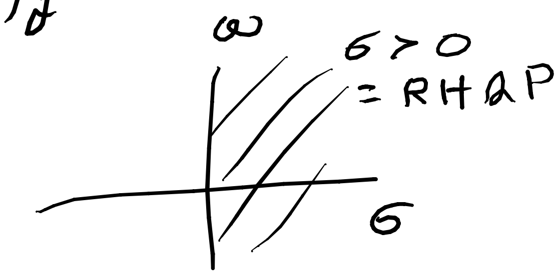
$$R\quad Y = (1_n+S)^{-1}(1_n-S) = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & 2 \\ 2 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \qquad Y = g\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}, \quad g = \frac{1}{r}.$$

$$Y = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -g \\ 0 & 0 & 0 \\ g & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & g \\ 0 & -g & 0 \end{bmatrix}$$



connect in parallel gives the circulator for terminating resistance $r = 1/g$



on to passivity:

1. real for real s, $\sigma > 0$
2. analytic in $\sigma > 0$
3. passive condition

Z or Y this is

$$\left.\begin{aligned} 0 \leq 2H_Z(s) &= Z(s)^{T*} + Z(s) \\ 0 \leq \text{ or } 2H_Y(s) &= Y(s)^{T*} + Y(s) \end{aligned}\right\} \text{in } \sigma > 0$$

positive semi definite Hermitian matrices

positive-real

if rational call PR

for S

$$H_S(s) = 1_n - S^{T*}(s)S(s) \geq 0 \text{ in } \sigma > 0$$

$* =$ complex conjugate

$j \to -j$

$j = \sqrt{-1}$

bounded-real

if rational call BR

Why condition 3

$$\mathcal{E}(t) = \int_{-\infty}^{t} v^{T*}(\tau) i(\tau) d\tau \geq 0 \text{ if passive for all } t$$

& all v, i supported by the circuit

$\omega = 2\pi f$

$$\mathcal{E}(\infty) = \int_{-\infty}^{\infty} v^{T*}(\tau)\, i(\tau)\, d\tau = \int_{-\infty}^{\infty} V^{T*}(j\omega)\, \underline{I}(j\omega)\, df$$ Parseval's relationship

$$V(j\omega) = \int_{-\infty}^{\infty} v(\tau)\, e^{-j 2\pi f \tau}\, d\tau$$ = Fourier transform of v

$$\mathcal{E}(\infty) \geq 0 \text{ if passive} = \int_{-\infty}^{\infty} V^{T*}(j\omega)\, Y(j\omega)\, V(j\omega)\, d\frac{\omega}{2\pi} \geq 0$$ (ie, real & not <0)

$$= \int_{-\infty}^{\infty} \frac{V^{T*}(j\omega)\, Y(j\omega)\, V(j\omega)}{2} + \left[\frac{V^{*T}(j\omega)\, Y^{T*}(j\omega)\, V^{\cancel{TT**}}(j\omega)}{2}\right] d\frac{\omega}{2\pi} \geq 0$$

implies that $V^{T*}\left[\underbrace{\frac{Y(j\omega) + Y^{T*}(j\omega)}{2}}\right] V(j\omega) \geq 0$, $\forall$ all V

(this is real) Hermitian matrix which is positive semi-definite

Note that $\int_{-\infty}^{\infty} v^{T*} i \, d\tau = \left(\int_{-\infty}^{\infty} v^{T*} i \, d\tau \right)^{T*} = \int_{-\infty}^{\infty} i^{T*} v \, d\tau$

$$\therefore \int_{-\infty}^{\infty} v^{T*} i \, d\tau = \int_{-\infty}^{\infty} \frac{v^{T*} i}{2} d\tau + \int_{-\infty}^{\infty} \frac{i^{T*} v}{2} d\tau$$

$$= \int_{-\infty}^{\infty} \frac{V^{T*} Y V}{2} \frac{d\omega}{2\pi} + \int_{-\infty}^{\infty} \frac{V^{T*} Y^{T*} V}{2} \frac{d\omega}{2\pi} = \int_{-\infty}^{\infty} V^{T*} \left[\frac{Y(j\omega) + Y^{T*}(j\omega)}{2} \right] V \frac{d\omega}{2\pi}$$

$\therefore$ by passivity $\int_{-\infty}^{\infty} V^{T*}(j\omega) \left[\frac{Y(j\omega) + Y^{T*}(j\omega)}{2} \right] V(j\omega) \frac{d\omega}{2\pi} \geq 0$ for all $V(j\omega) \in L_{2m}$

Choose $V(j\omega)$ = constant $\times$ narrow pulse at ω_0 for any and all ω_0 in which case, for all complex constant vectors V

$$0 \leq V^{T*} \left[\frac{Y(j\omega) + Y^{T*}(j\omega)}{2} \right] V = \mathrm{Re}\, V^{T*} Y(j\omega) V$$

it is the real part of $V^{T*} Y V$

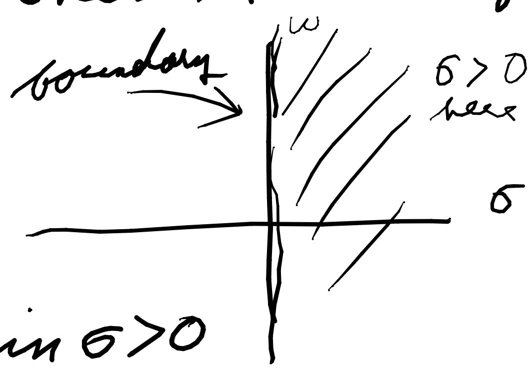
$$e^{-f(s)} = e^{-\mathrm{Re}\, f(s)} \cdot e^{-j\,\mathrm{Im}\, f(s)}\;; \quad |\cdot| = \text{absolute value}$$

$$\left|e^{-f(s)}\right| = \left|e^{-\mathrm{Re}\, f(s)}\right| \cdot 1$$

if $f(s)$ is analytic in $\sigma > 0$ then $e^{-f(s)}$ is also

(maximum modulus theorem) the maximum of $|e^{-f(s)}|$ occurs on a boundary of where it is analytic

Here $\mathrm{Re}\, V^{T*} Y(j\omega) V \geq 0$ on this boundary



$\therefore$ by maximum modulus theorem $\mathrm{Re}\, V^{T*} Y(s) V \geq 0$ in $\sigma > 0$

$\equiv \dfrac{Y(s) + Y^{T*}(s)}{2}$ is positive semidefinite in $\sigma > 0$

$\therefore$ $Y(s)$ for a finite passive circuit is PR

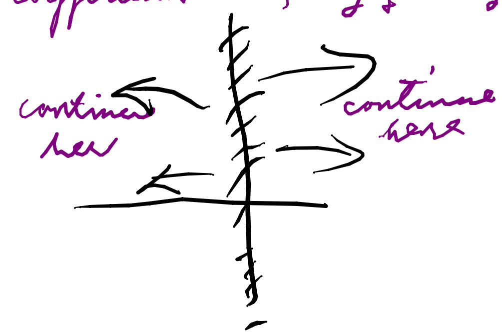
In the lossless case: $\mathcal{E}(\infty) = 0_{1\times 1}$ $\Leftrightarrow$ no energy remains in circuit

$$\Rightarrow \quad Y(j\omega) + Y^{T*}(j\omega) = 0_{n\times n}$$

$$\equiv \quad Y(j\omega) + Y^{T}(-j\omega) = 0_{n\times n}$$ if rational

(all coefficients real, only j is on $j\omega$)

an analytic continuation



let $\omega \to s/j$ $\Rightarrow$ can now use for all complex s

$$Y(s) + Y^{T}(-s) = 0_{n\times n}$$

true everywhere in the s-plane

Example:

$$\Rightarrow \quad Y(s) = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}$$

$Y(s) + Y^T(-s)$:

$$Y^T(-s) = \begin{bmatrix} -sC & sC - g \\ sC + g & -sC \end{bmatrix}$$

add $Y + Y_*^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow$ the Richards' extraction section is lossless

Grader: S. Raut $\Rightarrow$ sraut@umd.edu