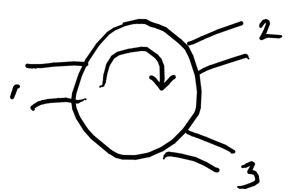
EE610
09/23/10

3-port circulator



$$v^r = \begin{bmatrix} v_1^r \\ v_2^r \\ v_3^r \end{bmatrix} = S v^i = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \\ v_3^i \end{bmatrix}$$

$$b = Sa$$

$$S_{circ} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad 1_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 1_3 - S_{circ}^T S_{circ} = 1_3 - \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = 0_3$$

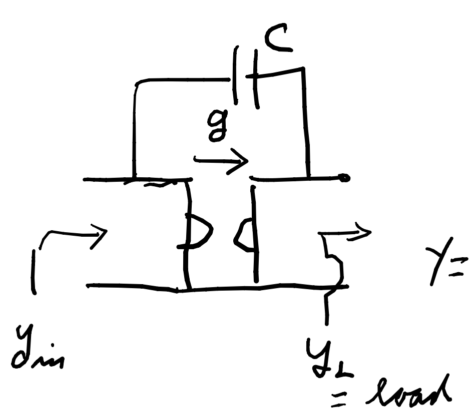
$$S_{circ}^{-1} = S_{circ}^T$$

Next Richards' functions ⇒ gives a positive real function of a positive real function

immitance = Y or a Z

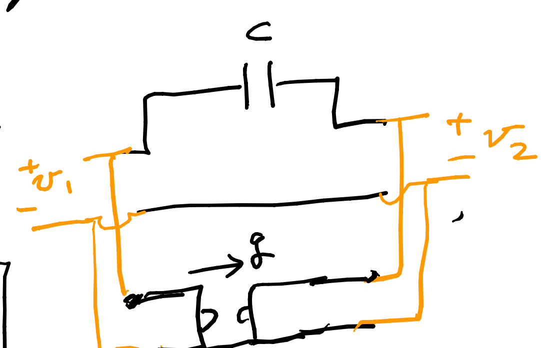
for a passive circuit an immitance is positive real

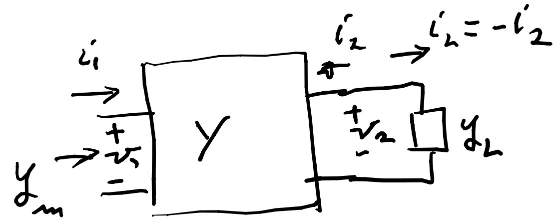
we will use for passive synthesis



$$Y_{gm} + Y_c = Y$$

$$Y = \begin{bmatrix} Cs & -Cs + g \\ -Cs - g & Cs \end{bmatrix}$$





$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\underset{\underset{-y_L V_2}{\shortparallel}}{-I_L} = I_2$$

$$-y_L V_2 = y_{21} V_1 + y_{22} V_2 \Rightarrow V_2 = -(y_L + y_{22})^{-1} y_{21} V_1$$

$$I_1 = y_{11} V_1 + y_{12} V_2 = y_{11} V_1 - y_{12}(y_L + y_{22})^{-1} y_{21} V_1$$

$$I_1 = y_{in} V_1$$

$$\Rightarrow y_{in} = y_{11} - y_{12}(y_L + y_{22})^{-1} y_{21}$$

as y_{22} is a scalar for Y 2×2,

determinant

$\Downarrow$

$$y_{in} = \frac{y_{11} y_L + y_{11} y_{22} - y_{12} y_{21}}{y_L + y_{22}} = \frac{y_{11} y_L + \Delta_Y}{y_L + y_{22}}$$

here $\begin{vmatrix} Cs & -Cs+g \\ -Cs-g & Cs \end{vmatrix} = (Cs)^2 - (-Cs+g)(-Cs-g) = g^2$

for our C "loaded" gyrator

$$y_{in} = \frac{Cs\, y_L + g^2}{y_L + Cs} \Rightarrow y_{in} \cdot y_L + y_{in} Cs = Cs\, y_L + g^2$$

$$(y_{in} - Cs) y_L = g^2 - y_{in} Cs$$

$$y_L = \frac{g^2 - Cs\, y_{in}}{y_{in} - Cs} = \frac{(g^2/C) - s\, y_{in}(s)}{(\frac{1}{C}) y_{in}(s) - s}$$

see p. 361 for a Richards' function

$$R(s) = \frac{k\, Z(s) - s\, Z(k)}{k\, Z(k) - s\, Z(s)}$$

$$R_2(s) = \frac{k\, y(k) - s\, y(s)}{k\, y(s) - s\, y(k)}$$

change $R_2(s)$ into $y(k)R_2(s) = \dfrac{k\,y(k) - s\,y(s)}{\frac{k}{y(k)}\cdot y(s) - s}$

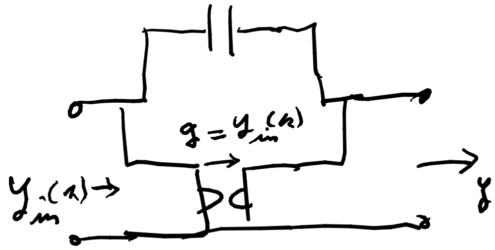
indentify y_L with $y(k)R_2(s)$

$$y = y_{in} \quad 1)$$
$$k\,y(k) = g^2/c \quad 2)$$
$$\frac{k}{y(k)} = \frac{1}{c} \quad 3)$$

$\overset{3)}{\Longrightarrow}$ $c = y_{in}(k)/k$ $\qquad$ $\overset{2)}{\Longrightarrow}$ $k\,y(k) = g^2/c$

$$= g^2 \cdot \frac{k}{y(k)} \Rightarrow g^2 = y_{in}^2(k)$$

$c = y_{in}(k)/k$



choose k = zero of even part of $y_{in}(s)$ then

$$\delta[y_L] = \delta[y_{in}] - 1$$
$\;\,\|$
degree

$2\,Ev\,y(s) = y(s) + y(-s)$ $\quad$ if $s = k$ is a zero of this $y(k) = -y(-k)$

$$y_L(s) = y_{in}(k)\left[\frac{k\,y_{in}(k) - s\,y_{in}(s)}{k\,y_{in}(s) - s\,y_{in}(k)}\right] = y_{in}(k)\left[\frac{k\,y_{in}(k) - (-k)\,y_{in}(-k)}{k\,y_{in}(-k) - (-k)\,y_{in}(k)}\right]$$

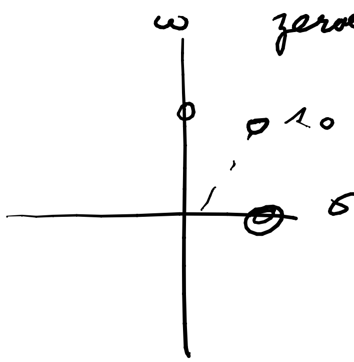
evaluate at $s = -k$ $\qquad$ $\underset{\|}{-y_{in}(k)}$

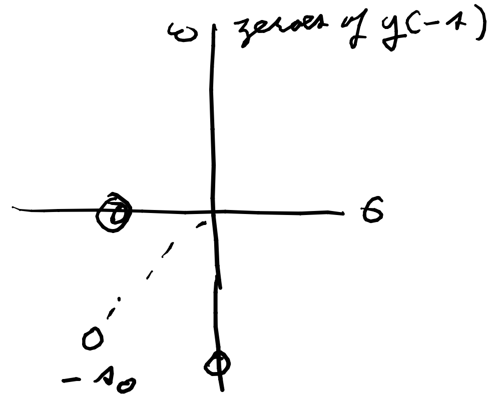
$$= y_{in}(k)\left[\frac{k\,y_{in}(k) + k\,y_{in}(-k)}{0}\right]$$

$= 0/0$

$\therefore$ $s - k$ & $s + k$ both cancel numerator & denominator

$\therefore$ choose k = a zero of $Ev\,y_{in}(s) = \dfrac{y_{in}(s) + y_{in}(-s)}{2}$





$$P_{ave} = \mathrm{Re}\, V^{T*}(j\omega)\, I(j\omega) = \mathrm{Re}\left[V^{T*}(j\omega)\left[y(j\omega)\, V(j\omega)\right]\right]$$

SSS = sinusoidal steady state

$$= \mathrm{Re}\; y(j\omega)\cdot V^{*}(j\omega) V(j\omega) \quad \text{for a 1-port}$$

$$= \frac{1}{2}\left(y(j\omega) + y(-j\omega)\right) |V|^2$$

↑ if real coeff $y^{*}(j\omega)$

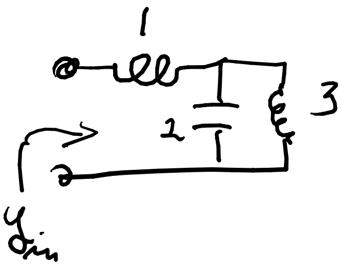
if lossless $P_{ave}(j\omega) = 0$

$$y(j\omega) + y(-j\omega) \equiv 0 \quad \text{for almost all } \omega$$

this has $y(j\omega)$ analytic in $\sigma > 0$ so can extend $y(j\omega)$ to $\sigma > 0$ by $\omega = s/j$

$$y(s) + y(-s) = 0 \quad \text{identically}$$

Ex:



$$\Rightarrow y_{in} = \frac{1}{z_{in}} = \frac{1}{s + \frac{1}{2s + \frac{1}{3s}}} =$$

$$= \frac{1}{s + \frac{3s}{6s^2+1}} = \frac{6s^2+1}{6s^3+4s}$$

$$= \frac{1}{s}\left(\frac{6s^2+1}{6s^2+4}\right)$$

$$y_{in}(s) + y_{in}(-s) = \frac{1}{s}\left(\frac{6s^2+1}{6s^2+4}\right) - \frac{1}{s}\left(\frac{6s^2+1}{6s^2+4}\right) \equiv 0$$

$\therefore$ any complex k is a zero of $Ev\, y_{in}(s)$

$$C = \frac{y(k)}{k} = \frac{1}{k^2}\left(\frac{6k^2+1}{6k^2+4}\right)$$

choose $k = 2$ is a real zero of $Ev\, y(s) \Rightarrow C = \frac{1}{4}\left(\frac{25}{28}\right)$

$$g = y(k) = \frac{1}{2}\left(\frac{25}{28}\right)$$

$$y_L = y(k)\,\frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$$

we know that the polynomial $(s-k)(s+k) = s^2 - k^2 = s^2 - 4$ cancels in numerator & denominator

$$= \frac{1}{2}\left(\frac{25}{28}\right)\left[\frac{2\left(\frac{1}{2}\right)\left(\frac{25}{28}\right) - \frac{6s^2+1}{6s^2+4}}{2\,\frac{1}{s}\left(\frac{6s^2+1}{6s^2+4}\right) - \frac{s}{2}\left(\frac{25}{28}\right)}\right] = \frac{1}{2}\frac{25}{28}\left[\frac{\frac{25}{28}(6s^2+4) - (6s^2+1)}{\frac{2}{s}(6s^2+1) - \frac{s}{2}\left(\frac{25}{28}\right)(6s^2+4)}\right]$$

$$= \frac{1}{2}\frac{25}{28}s\left[\frac{6\left(\frac{25}{28}-1\right)s^2 + \left(\frac{25}{28}\cdot 4 - 1\right)}{-3\left(\frac{25}{28}\right)s^4 + 12s^2 - 2\left(\frac{25}{28}\right)s^2 + 2}\right]$$

$$= \frac{1}{2}\left(\frac{25}{28}\right)s\left[\frac{-\frac{18}{28}s^2 + \frac{72}{28}}{-\frac{75}{28}s^4 - \frac{236}{28}s^2 + \frac{56}{28}}\right] = \frac{-\frac{18}{28}(s^2-4)s}{\frac{(s^2-4)(-1)(75s^2+4)}{28}}$$

$$\begin{array}{r} -75s^2 - 4 \\ s^2-4 \,\big)\, \overline{-75s^4 + 286s^2 + 56} \\ -75s^4 + 300s^2 \\ \hline -14s^2 + 56 \end{array}$$

$$y_L = \frac{18}{28}s\,\frac{1}{(75s^2+4)/28} = \frac{18s}{75s^2+4}$$

100
−27

$12 - \frac{50}{28}$

12
56
18

300
−28