EE 610
09/21/10 b

$S(s)$ is bounded-real; S $n \times n$
if the circuit passive

real 1. $S(\sigma)$ is real for $\sigma > 0$ (real coefficients if ratios of polynomials)

stable 2. $S(s)$ is analytic in $\sigma > 0$
& no poles on $\sigma = 0$

passive 3. $1_n - S^{T*}(s)S(s)$ is positive semi-definite in $\sigma \geq 0$; $s = \sigma + j\omega$

$$\Rightarrow a^{T*}\left(1_n - S^{T*}(s)S(s)\right)a \geq 0$$ for all complex a vectors

Example: $C > 0$ $S(s)$

$i(t) = C\dfrac{dv}{dt} \Rightarrow I(s) = C\,s\,V(s)$;

$a = \dfrac{v + ri}{2r} = \dfrac{v^i}{r}$

$b = \dfrac{v - ri}{2r} = \dfrac{v^r}{r}$

$B(s) = S(s)A(s)$

$v = v^i + v^r$

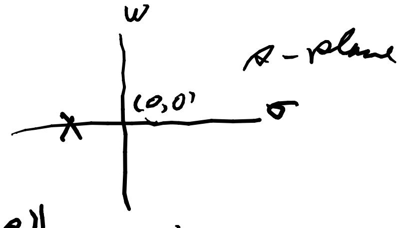
$i = v^i - v^r$

$V^r = S(s)V^i$

$V^i - V^r = Cs(V^i + V^r)$

$-V^r - CsV^r = CsV^i - V^i$

$V^r = \dfrac{1}{1+Cs}(1 - Cs)\cdot V^i \Rightarrow S(s) = \dfrac{1 - Cs}{1 + Cs}$



1) true $\Rightarrow$ coeff real

2) true $\Rightarrow$ pole at $s = -1/C$

3) $1 - \left(\dfrac{1 - Cs^*}{1 + Cs^*}\right)\left(\dfrac{1 - Cs}{1 + Cs}\right) = 1 - \dfrac{|1 - Cs|}{|1 + Cs|}$ in $s = \sigma + j\omega$ where $\sigma > 0$

$$= 1 - \left|\frac{1 - C\sigma - jC\omega}{1 + C\sigma + jC\omega}\right| = 1 - \frac{\left((1 - C\sigma)^2 + C^2\omega^2\right)^{1/2}}{\left((1 + C\sigma)^2 + C^2\omega^2\right)^{1/2}} > 0$$

Note on $S(j\omega)$: For the capacitor $S(j\omega) = \dfrac{1-j\omega c}{1+j\omega c}$;

$$S^*(j\omega) = S(-j\omega) = \frac{1+j\omega c}{1-j\omega c} = \frac{1}{S(j\omega)}$$

$\therefore\ 1 - S^*(j\omega)S(j\omega) = 1 - 1 = 0$ for the capacitor

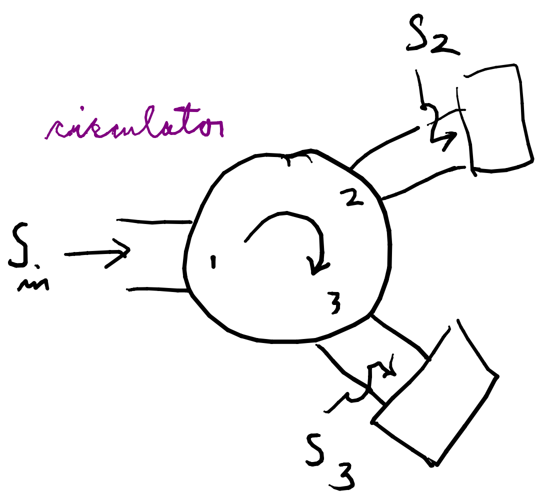
and $S(-s)\big|_{s=j\omega} = S^*(j\omega)$

poles of Y & Z can be on $j\omega$ axis:

$$\underbrace{Y(s) = sC}_{\text{pole at } \infty},\quad \underbrace{Z(s) = \frac{1}{sC}}_{0}$$ on $j\omega$ axis

but for $S(s)$ example: $S(s) = \dfrac{1-sC}{1+sC}$ no pole in $\sigma \geq 0$ if $C > 0$

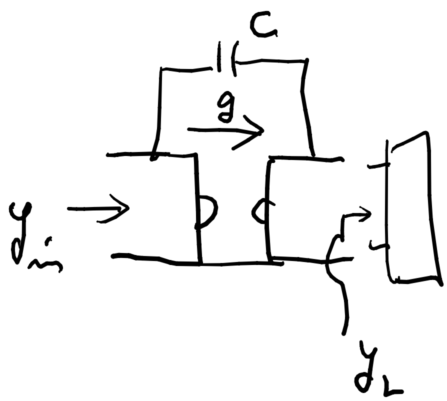
circulator



$S_{in} = S_3 \cdot S_2$ only if same r

Note: $V^r_{2_{circ}} = V_{2_{circ}} - rI_{2_{circ}} = V_{2_{load}} + rI_{load} = V^i_{2_{load}}$

(here $I_{2circ} = -I_{load}$)



Richards' p. 361 function

y_L is related to y_{in} via the Richards' function (times $y_{in}(s)$)