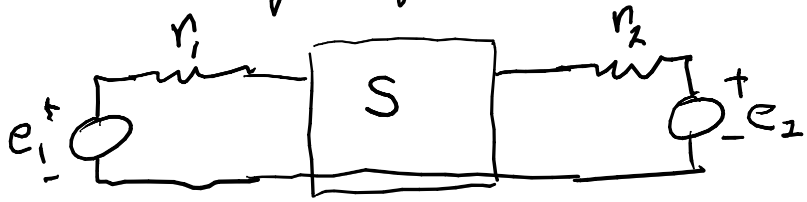
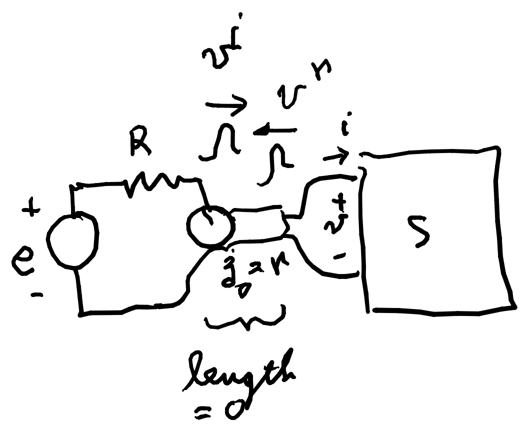
EE 610
09/16/10 → 09/17/10

Scattering matrix
(useful for microwaves, wireless)



$R = r\mathbb{1}_2$

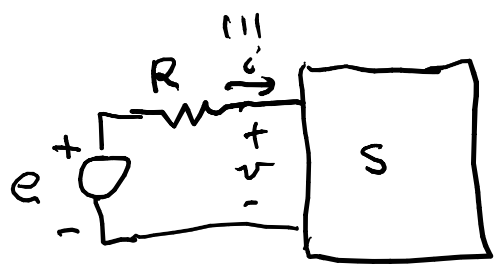
$r = r_1 = r_2$



$v^r = S v^i$

$e = v + Ri = 2\sqrt{R}\,a = 2v^i$

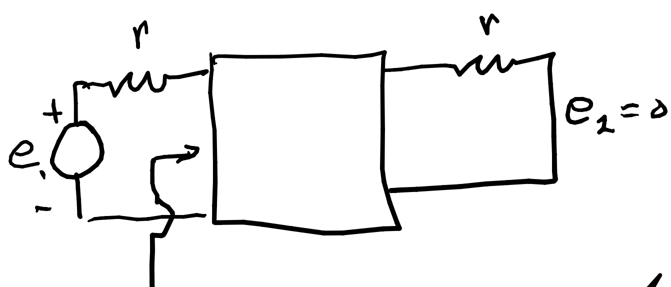
$v - Ri = 2\sqrt{R}\,b = 2v^r$



$b = Sa \equiv v^r = S v^i$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{v_1 - r i_1}{(2\sqrt{r})\left(\frac{v_1 + r i_1}{2\sqrt{r}}\right)} = \frac{v_1 - r i_1}{v_1 + r_1 i_1}$$

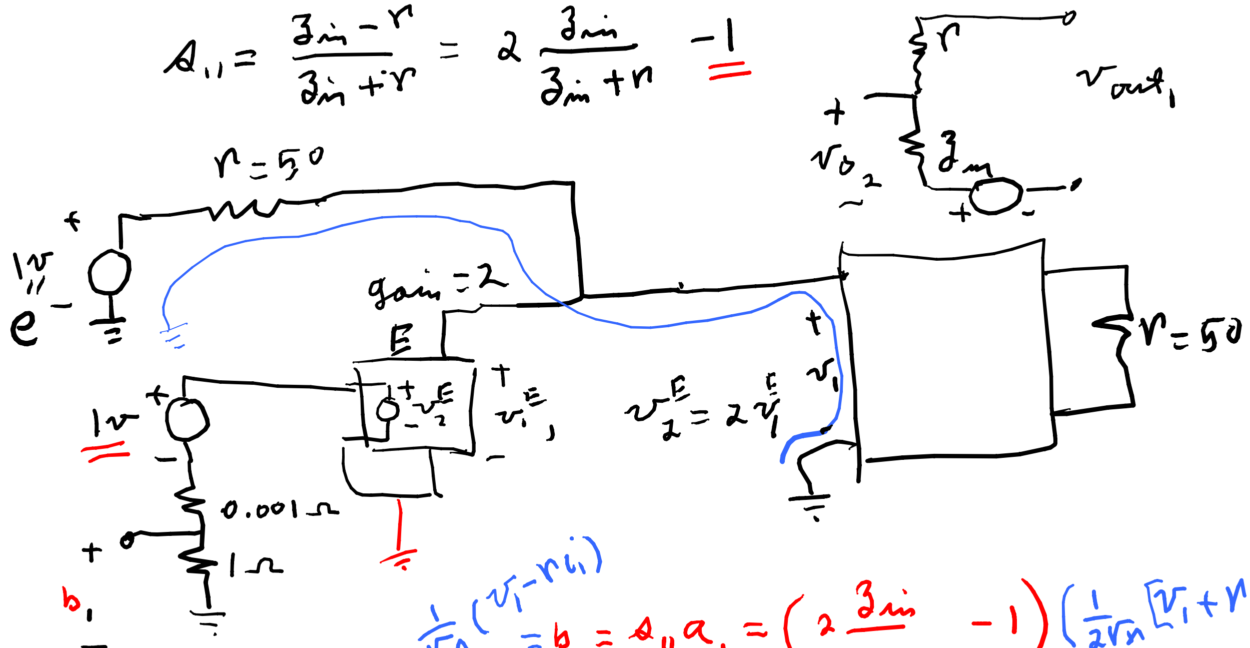


$v_1 = z_{in} i_1 \Big|_{e_2=0}$

if z_{in} for the loaded 2-port exists

$$s_{11} = \frac{(v_1/i_1 - r)\, i_1}{(v_1/i_1 + r)\, i_1} = \frac{z_{in} - r}{z_{in} + r}\,; \quad r = z_0$$

$$A_{11} = \frac{z_{in} - r}{z_{in} + r} = 2\frac{z_{in}}{z_{in}+r} - 1$$



note: add the ground to E 09/17 corrects app note

$$\frac{1}{2\sqrt{r}}(v_1 - r i_1) = b_1 = A_{11} a_1 = \left(2\frac{z_{in}}{z_{in}+r} - 1\right)\left(\frac{1}{2\sqrt{r}}[v_1 + r i_1]\right)$$

$\Downarrow$ $v_1 - R i_1$ $\qquad$ $\Downarrow$ $e = 1v$

gives A_{11} as the voltage on the 1Ω resistor

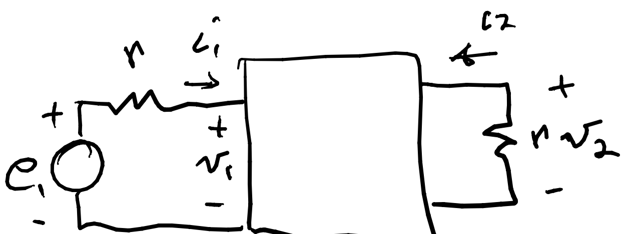
since the input to E is $v_{on\, z_{in}}$ and the output at b_1 is $2v_{on\, z_{in}} - 1$ for 1v for e ⇒ $v_{on\, z_{in}} = \frac{z_{in}}{r+z_{in}} \times 1$

⇒ v for b_1 is $2\frac{z_{in}}{r+z_{in}} \times 1 - 1$

09/17

For A_{21}: $\quad A_{21} = \left.\frac{b_2}{a_1}\right| = \frac{v_2^r}{v_1^i}$

$$a_2 = 0 = v_2^i$$



$$2v_2^r = v_2 - r i_2 = 2v_2$$

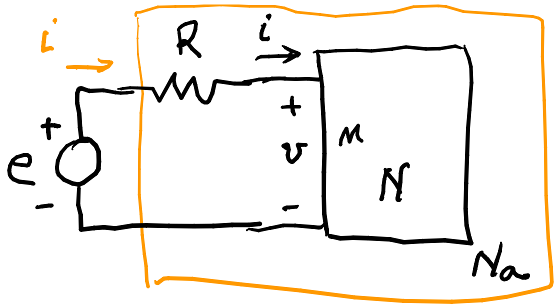
but $v_2 = -r i_2$

$$e_1 = v_1 + r i_1 = 2v_1^i$$

$$A_{21} = \frac{v_2^r}{v_1^i} = \frac{\cancel{2}\,v_2}{e_1/2} = \cancel{4}\,2\,\frac{v_2}{e_1} = 2\times \text{terminated voltage gain}$$

Reference Cadence application note:
http://www.cadence.com/rl/Resources/application_notes/S-Parameter_Data_appnote.pdf

needs ground on E component & $\sqrt{z}$ in eqs. (1)-(4) is best as $\sqrt{Z_0}$ or even set to 1. [E component of Spice = voltage controlled voltage source]



$$e \Rightarrow i = Y_a e$$

$$\left.\begin{aligned} e &= v + Ri \\ &= v + R Y_a e \end{aligned}\right\} \Rightarrow (1_m - R Y_a) e = v$$

$$Y_a e = i$$

Choose $R = r 1_m \Rightarrow R Y_a = Y_a R$

$$Y_a (1_m - R Y_a) e = Y_a v$$

$$(1_m - R Y_a) Y_a e = (1_m - R Y_a) i$$

$$(Y_a - Y_a R Y_a) e$$

$$(Y_a - R Y_a Y_a) e = (Y_a - Y_a R Y_a) e \Rightarrow Y_a v = (1_m - R Y_a) i$$

$$Av = Bi \Rightarrow A = Y_a, \; B = 1_m - R Y_a = 1_m - Y_a R$$

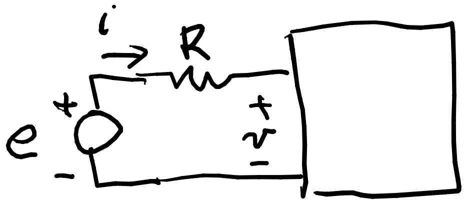
But $\left.\begin{aligned} 2v^i &= v + Ri \\ 2v^r &= v - Ri \end{aligned}\right\}$ $\left.\begin{aligned} 2v &= 2v^i + 2v^r \\ 2Ri &= 2v^i - 2v^r \end{aligned}\right\}$ $\begin{aligned} v &= v^i + v^r \\ Ri &= v^i - v^r \end{aligned}$

$$Av = Bi \Rightarrow A(v^i + v^r) = (BR^{-1})(\underbrace{v^i - v^r}_{Ri})$$

$$(A - BR^{-1}) v^i = -(A + BR^{-1}) v^r$$

$$v^r = (BR^{-1} + A)^{-1} (BR^{-1} - A) v^i$$

$$\begin{aligned} S &= (BR^{-1} + A)^{-1} (BR^{-1} - A) \\ &= (R^{-1} - R Y_a R^{-1} + Y_a)^{-1} (R^{-1} - R Y_a R^{-1} - Y_a) \\ &= R(R^{-1} - 2Y_a) = 1_m - 2 R Y_a \end{aligned}$$



$$e = v + Ri$$

$$p(t) = v^T(t)\, i(t)$$

$$e^T(t)e(t) = (v+Ri)^T(v+Ri) = v^T(t)v(t) + i^T R v + v^T R i + i^T R R i$$

$$(i^T R v)^T = i^T R v = v^T R i$$

$$\underbrace{e^T e}_{\Sigma \text{ of sqs.}} = \underbrace{v^T v}_{\Sigma \text{ of sqs}} + \underbrace{(Ri)^T}_{\text{sum of sqs.}} R i + 2\underbrace{v^T R i}_{2r\, \underbrace{v^T i}_{p(t)}}$$

$$\underbrace{\int_{-\infty}^{t} e^T(\tau)e(\tau)\,d\tau}_{\text{if finite}} = \underbrace{\int_{-\infty}^{t} v^T(\tau)v(\tau)\,d\tau + \int_{-\infty}^{t} (Ri)^T R i}_{} + 2r \underbrace{\int_{-\infty}^{t} p(\tau)\,d\tau}_{\text{if passive } \geq 0}$$

$\Rightarrow e_i \in L_2$ $\qquad \Rightarrow v_i \in L_2$ $\qquad \Rightarrow i_i \in L_2$

$\Rightarrow$ eventually $\Rightarrow$ $S(s)$ is bounded-real for a passive N