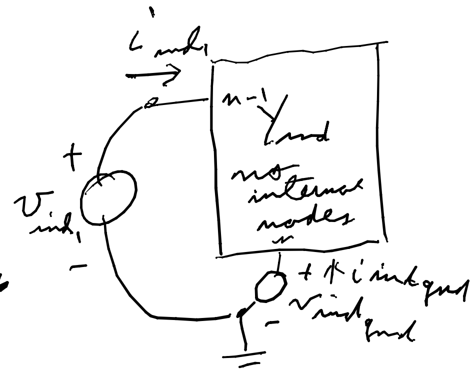
EE 610
09/14/10

Indefinite admittance
Nodal ''
Terminal ''
Loaded ''

Indefinite $Y_{ind}(s)$



$$\begin{bmatrix} i_{ind_1} \\ \vdots \\ i_{ind_{gnd}} \end{bmatrix} = i = Y_{ind} v$$

v: node voltages w.r.t gnd

$\sum_{k=1}^{n} i_k = 0 \Rightarrow$ rows of Y_{ind} sum to zero if put $1v$ on every node one at a time

$\Rightarrow$ columns of Y_{ind} sum to 0

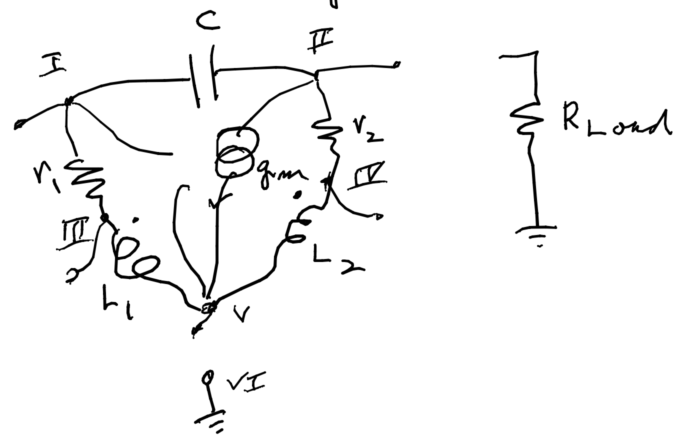
add the same voltage to every node voltage $E = \begin{bmatrix} e_1 \\ e_1 \\ \vdots \\ e_1 \end{bmatrix}$ an n vector

$i = Y_{ind}(v + E)$ for any E get the same i

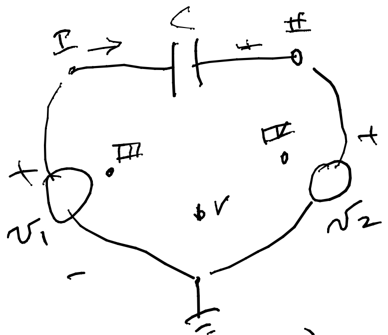
$Y_{ind} E = \underline{0} \Rightarrow$ all entries in a row sum to 0

rows & columns of Y_{ind} sum to zero

Ex:

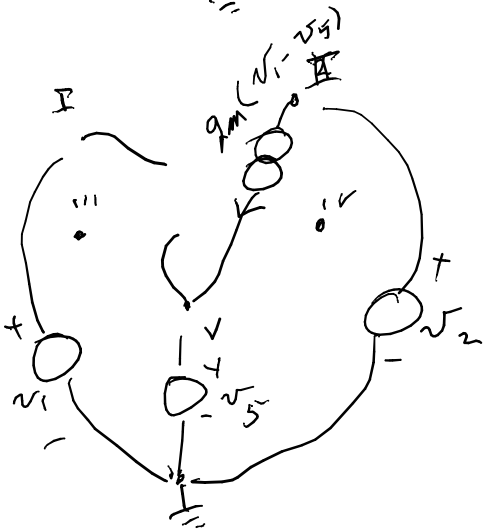


$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_5 \end{bmatrix} = \begin{bmatrix} v_I \\ \vdots \\ v_V \end{bmatrix}$$



$$i_{cap} = C\frac{dv_{cap}}{dt}$$

$$\text{Cap: } Y = \begin{array}{cc} & \begin{array}{ccccc} 1 & & & & \end{array} \\ 1 & \begin{bmatrix} sC & -sC & 0 & 0 & 0 \\ -sC & sC & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ g_m & 0 & 0 & 0 & -g_m \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -g_m & 0 & 0 & 0 & g_m \end{bmatrix}$$

$$Y_{ind} = \begin{bmatrix} Cs+g_1 & -Cs & -g_1 & 0 & 0 \\ g_m - Cs & Cs+g_2 & 0 & -g_2 & -g_m \\ -g_1 & 0 & g_1+\frac{1}{sL_1} & 0 & -\frac{1}{sL_1} \\ 0 & -g_2 & 0 & g_2+\frac{1}{sL_2} & -\frac{1}{sL_2} \\ -g_m & 0 & -\frac{1}{sL_1} & -\frac{1}{sL_2} & g_m+\frac{1}{sL_1}+\frac{1}{sL_2} \end{bmatrix}$$

Move the ground to the Hartley: $\Rightarrow v_5 = 0 \Rightarrow$ can remove the last column

can remove any row, as $\sum i's = 0$ by KCL

choose the 5th as it is ground

gives nodal admittance, 4×4

$$Y_{node} = \left[\begin{array}{cc|cc} Cs+g_1 & -Cs & -g_1 & 0 \\ g_m - Cs & Cs+g_2 & 0 & -g_2 \\ \hline -g_1 & 0 & g_1+\frac{1}{sL_1} & 0 \\ 0 & -g_2 & 0 & g_2+\frac{1}{sL_2} \end{array}\right] = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right]$$

$$\begin{bmatrix} i_1 \\ i_2 \\ \hline 0 \\ 0 \end{bmatrix} = Y_{node} \begin{bmatrix} v_1 \\ v_2 \\ \hline v_3 \\ v_4 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} V_{12} \\ V_{34} \end{bmatrix}$$

$$I_1 = Y_{11} V_{12} + Y_{12} V_{34}$$

$$0 = Y_{21} V_{12} + Y_{22} V_{34}$$

desire to eliminate V_{34}; ie ignore the internal nodes

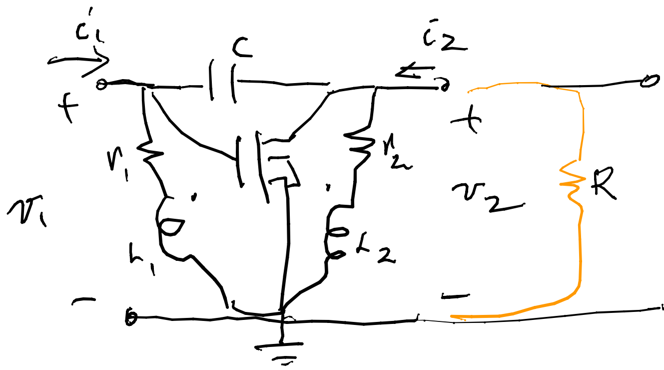
$$\Rightarrow V_{34} = -Y_{22}^{-1}(Y_{21} V_{12})$$

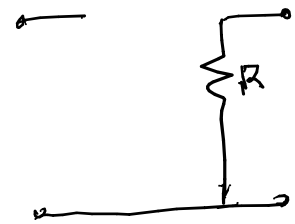
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = I_1 = (Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}) V_{12} = Y_{term} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{term} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$= \begin{bmatrix} Cs+g_1 & -Cs \\ g_m - Cs & Cs+g_2 \end{bmatrix} - \begin{bmatrix} -g_1 & 0 \\ 0 & -g_2 \end{bmatrix} \begin{bmatrix} \frac{sL_1}{1+sg_1L_1} & 0 \\ 0 & \frac{sL_2}{1+sg_2L_2} \end{bmatrix} \begin{bmatrix} -g_1 & 0 \\ 0 & -g_2 \end{bmatrix}$$

$$Y_{term} = Y_{2\text{-port}} = \begin{bmatrix} Cs+g_1 - \frac{g_1^2 sL_1}{1+sg_1L_1} & -Cs \\ g_m - Cs & Cs+g_2 - \frac{sL_2}{1+sg_2L_2} \end{bmatrix}$$





$\Rightarrow Y_{\text{load}} = \begin{bmatrix} 0 & 0 \\ 0 & G \end{bmatrix}$

$G = 1/R$

Can load by adding admittances

$$Y_{\substack{\text{2-port} \\ \text{with load}}} = \begin{bmatrix} Cs + g_1 - \dfrac{g_1^2 sL_1}{1 + sg_1L_1} & -Cs \\ g_m - Cs & Cs + g_2 - \dfrac{sL_2 g_2^2}{1 + sg_2L_2} + G \end{bmatrix}$$

$$I = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y_{\substack{\text{2 port} \\ \text{load}}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Longrightarrow I = 0 \text{ when oscillate}$$

$$YV = \underline{0} \,; \quad V \neq 0$$

desire $\det Y = 0$; $Z = Y^{-1}$; $Y^{-1} I = V$; $I = 0$

as desire $Z \to \infty$ as $\infty \times 0$ can be $\neq 0$

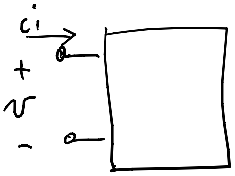
Now get degree 4 in s, $s = j\omega$, req'd $\operatorname{Re} \det Y(j\omega) = 0$

$\operatorname{Im} \det Y(j\omega) = 0$

If desire $\left.\dfrac{i_1}{v_1}\right|_{v_2 = 0} = y_{11} = Cs + \dfrac{g_1}{1 + sg_1L_1} = \dfrac{g_1 + Cs + s^2 C g_1 L_1}{1 + sg_1L_1}$

$\left.\dfrac{i_2}{v_1}\right|_{v_2 = 0} = y_{21} = g_m - Cs$

Passivity



$$p_{in}(t) = v^T(t)\, i(t)$$

$i_1 = 0$
$i_2 = g_m v_1 + g_0 v_2$

$$p_{in} = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \end{bmatrix} Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ g_m & g_0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \begin{bmatrix} v_2 g_m , & v_2 g_0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_1 v_2 g_m + v_2^2 g_0$$

$$\mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\, d\tau = \int_{-\infty}^{t} \left[g_m v_1(\tau) v_2(\tau) + v_2^2(\tau) g_0 \right] d\tau$$

want $\mathcal{E}(t) \geq 0$ for a passive device

here if $g_0 > 0$ the last term is > 0

but even if $g_m > 0$ the 1st term can have any sign & be as large as desired by the free choice of port voltage