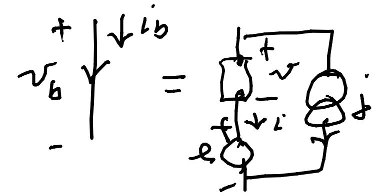
EE 610
09/09/10

$$x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} \; ; \quad v_b = \begin{bmatrix} v_t \\ v_\ell \end{bmatrix} = \mathcal{C}^T v_t \, , \quad 0_\ell = \mathcal{J} v_b$$

$$i_b = \begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = \mathcal{J}^T i_\ell \, , \quad 0_t = \mathcal{C} i_b$$

$v_b = v + e$
$i_b = i + j$



LLT
$$A(s) v = B(s) i$$

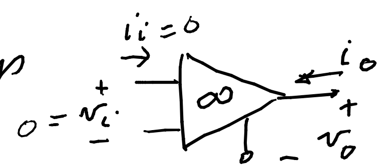
$$A v_b - A e = B i_b - B j$$

$$A \mathcal{C}^T v_t - B \mathcal{J}^T i_\ell = A e - B j \Rightarrow \left[A\mathcal{C}^T \;\vdots\; -B\mathcal{J}^T \right] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = A e - B j$$

$$\underbrace{\begin{aligned} s E x - \mathcal{A} x &= \mathcal{B} u \\ y &= \mathcal{C} x \end{aligned}}_{\text{universal description}}$$

; u = inputs from e, j
& $\mathcal{B}$ is constant by choice of separate branches for sources
$\mathcal{A}$ is constant,
y = outputs; assumed as linear combinations of voltages & currents i.e. in terms of x

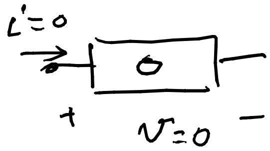
Note: op-amp



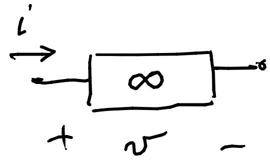
description is $v_i = 0$, $i_i = 0$
i_o & v_o free

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_i \\ v_o \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_i \\ i_o \end{bmatrix}$$

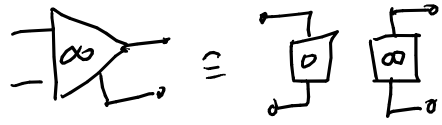
also



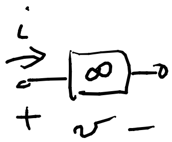
nullator



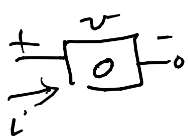
norator



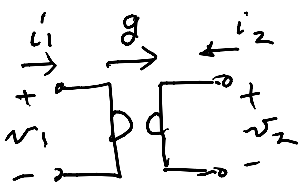
i & v free



$$Av = Bi \Rightarrow \begin{bmatrix} 0 \end{bmatrix}[v] = \begin{bmatrix} 0 \end{bmatrix}[i]$$



$$Av = Bi \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}[v] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}[i]$$



gyrator

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad I = YV$$

g = gyration conductance

$$Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} = -Y^T$$

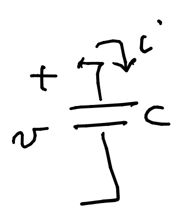
Power into the gyrator

$$= V^T I = V^T Y V = [v_1 \ \ v_2] \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= [-g v_2 \ \ g v_1] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -g v_1 v_2 + g v_1 v_2 = 0$$

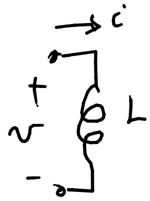
$\Updownarrow$

(instantaneously) lossless

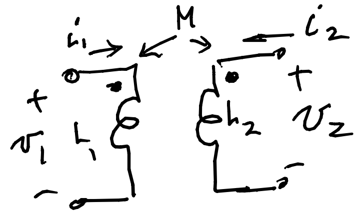


$$i = C\frac{dv}{dt} = \frac{dq}{dt}, \quad q = Cv$$

C = constant



$$v = L\frac{di}{dt}$$



$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = s\begin{bmatrix} l_{11} & l_{12} \\ l_{12} & l_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}; \quad V = sLI = ZI$$

$$Y = Z^{-1} = \frac{1}{det}\begin{bmatrix} sl_{22} & -sl_{12} \\ -sl_{12} & sl_{11} \end{bmatrix}; \quad det = sl_{11}\cdot sl_{22} - sl_{12}\cdot sl_{12} = s^2(l_{11}l_{22} - l_{12}^2)$$

coefficient of coupling $k^2 = \frac{l_{12}^2}{l_{11}l_{22}}$; $k = \frac{|l_{12}|}{\sqrt{l_{11}l_{22}}}$

if passive, $k \le 1$; $k = 1 \Rightarrow$ perfect coupling

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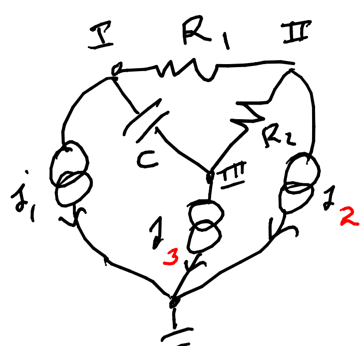
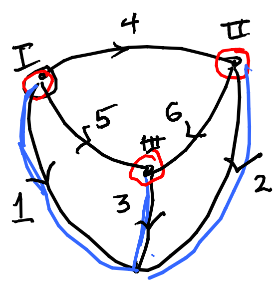
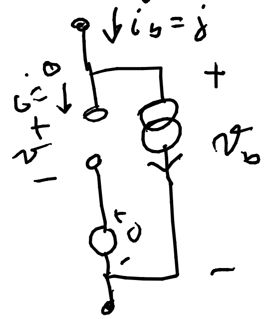
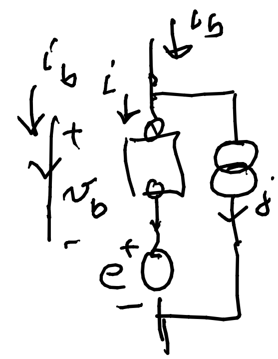
admittance $\Rightarrow$ nodal, terminal, indefinite



$\Rightarrow$







$Av = Bi \Rightarrow$ general description

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & sC & 0 \\ 0 & 0 & 0 & 0 & 0 & G_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

$$Y_{b\times b}\, v = 1_6\, i \; ; \quad Y_{b\times b}\, v_b = i_b - j$$

$$i_b = i + \begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad v_b = v + \underline{0}_b \; ; \quad \mathcal{C} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array}\right] ; \quad K = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$= \mathcal{C}^T v_t$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline & K^T & \end{array}\right] \begin{bmatrix} v_{b_1} \\ v_{b_2} \\ v_{b_3} \end{bmatrix} \qquad i_b = \mathcal{T}^T i_c = \left[-K \;\vdots\; 1_c \right] i_c$$

$$Y_{b\times b} v_b = i_b - j = Y_{b\times b} \mathcal{C}^T v_t$$

$$\times \mathcal{C} \Rightarrow \quad \mathcal{C} Y_{b\times b} \mathcal{C}^T v_t = \underbrace{\mathcal{C} i_b}_{=\underline{0}_b} - \mathcal{C} j = -\mathcal{C} j$$

$$\left[1_t \,\middle|\, K \right] \left[\begin{array}{c|c} 0_t & 0 \\ \hline 0 & Y_{comp} \end{array}\right] \begin{bmatrix} 1_t \\ K^T \end{bmatrix} = -\left[1_t \,\middle|\, K \right] \begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ \hline 0 \end{bmatrix} \begin{array}{l} \}\, j_t \\ \}\, j_c = \underline{0}_c \end{array}$$

$$K Y_{comp} K^T v_t = -j_t \; ; \quad \text{txt eqn.}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} G_1 & 0 & 0 \\ 0 & sC & 0 \\ 0 & 0 & G_2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} sC & G_1 & 0 \\ -sC & 0 & G_2 \\ 0 & -G_1 & -G_2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} sC + G_1 & -G_1 & -sC \\ -G_1 & G_1 + G_2 & -G_2 \\ -sC & -G_2 & G_1 + G_2 \end{bmatrix} = Y_{ind}$$

has sums of row & column entries both zero

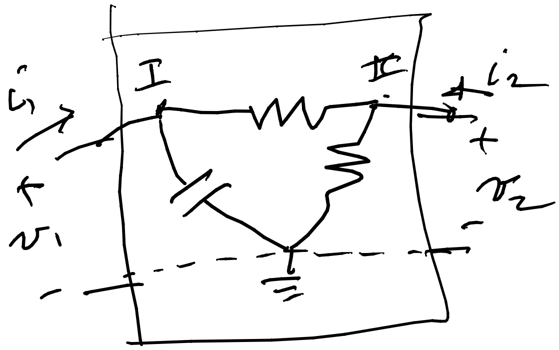
Move the ground to node 3 $\Rightarrow$ $v_3 = 0$

replaces 3rd col. by zero so can ignore

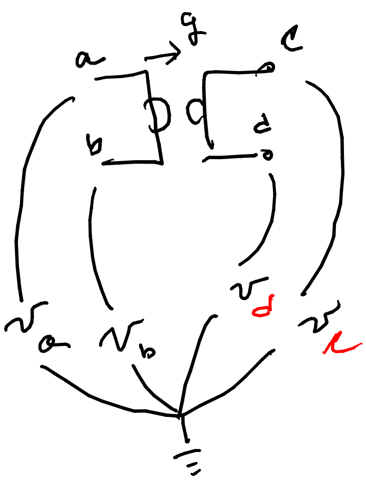
by KCL can ignore j_3

scratch out the 3rd row & column

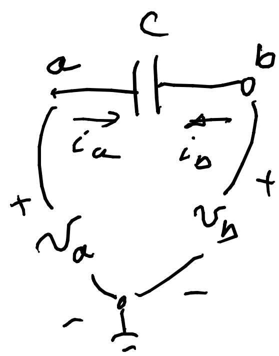
$$\begin{bmatrix} sC + G_1 & -G_1 \\ -G_1 & G_1 + G_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -j_1 \\ -j_2 \end{bmatrix}$$



$$Y_{2\text{-port}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = - \begin{bmatrix} j_1 \\ j_2 \end{bmatrix}$$



$$i_a = -i_b = g(v_c - v_d)$$



$$Y_{indep} = \begin{array}{cc} & \\ \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix} & \begin{array}{c} \leftarrow a \\ \leftarrow b \end{array} \\ \begin{array}{cc} \uparrow \; a & \quad \uparrow \; b \end{array} & \end{array}$$