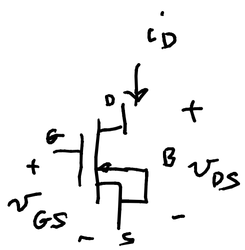
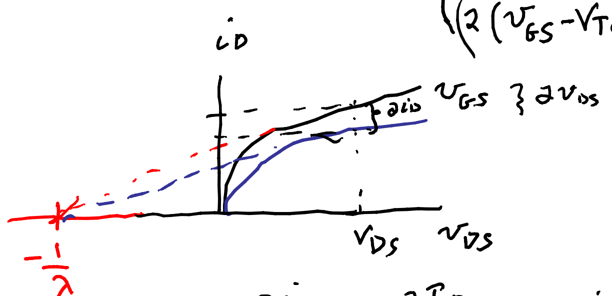
EE610
09/07/10

$$s\begin{bmatrix} C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_1 & 0 \\ 0 & 0 & 0 & 0 & -L_2 \end{bmatrix} x = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 \\ G_o & -(g_m+G_o) & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix} x$$



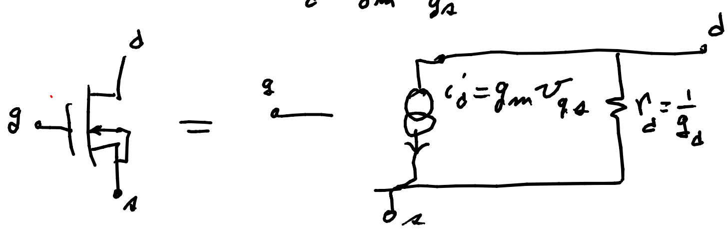
$$i_D = \frac{KP}{2}\cdot\frac{W}{L}\begin{cases} (v_{GS}-VTO)^2(1+\lambda v_{DS}) & \text{if } v_{GS} \geq VTO \ \& \ v_{DS} \geq v_{GS}-V_{TO} \\ 0 & \text{if } v_{GS} \leq VTO \\ \left(2(v_{GS}-VTO)v_{DS}-v_{DS}^2\right)(1+\lambda v_{DS}) & \text{if } v_{GS} \geq VTO, \ v_{DS} \leq v_{GS}-VTO \end{cases}$$



$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{\substack{v_{DS}=V_{DS} \\ (Q \text{ point})}} = \frac{2I_D}{(V_{GS}-V_{TO})} \quad \text{if in saturation}$$

$$\underset{\text{total}}{i_D} = \underset{\text{bias}}{I_D} + \underset{\text{signal}}{i_d} = I_D + \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q (\underbrace{v_{GS}-V_{GS}}_{v_{gs}}) + \cdots \text{ higher order}$$

$$i_d = g_m v_{gs}$$



VCCS

$$g_d = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q = \lambda\frac{I_D}{1+\lambda V_{DS}} \quad ; \quad s = \frac{d}{dt} \ ; \quad e^{st} \ ; \quad s = \sigma + j\omega$$

$$s\begin{bmatrix} C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_1 & 0 \\ 0 & 0 & 0 & 0 & -L_2 \end{bmatrix} x = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 \\ G_o & -(g_m+G_o) & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix} x \ ; \quad x = \begin{bmatrix} v_t \\ i_x \end{bmatrix}_b = \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b$$

semi-state

eliminate via 2nd row:

$$0 = -i_3 - i_4 - i_5 \Rightarrow i_3 = -i_4 - i_5$$

eliminate via 3rd row: $G_m = g_m + G_0$

$$0 = G_0 v_1 - G_m v_2 + i_3 \Rightarrow G_m v_2 = G_0 v_1 - i_4 - i_5$$

$$sCv_1 = sC x_1 = i_3 + i_5 = -i_4 - i_5 + i_5 = -i_4$$

$$-sL_1 i_4 = -sL_1 x_4 = -v_2 = -\frac{1}{G_m}\left[G_0 v_1 - i_4 - i_5\right] \qquad ; \quad R_m = 1/G_m$$

$$-sL_2 i_5 = -sL_2 x_5 = v_1 - v_2 = v_1 - \frac{G_0}{G_m} v_1 + \frac{1}{G_m} i_4 + \frac{1}{G_m} i_5$$

$$s\begin{bmatrix} C & 0 & 0 \\ 0 & L_1 & 0 \\ 0 & 0 & L_2 \end{bmatrix} \underline{x} = \begin{bmatrix} 0 & -1 & 0 \\ R_m G_0 & -R_m & -R_m \\ (R_m G_0 - 1) & -R_m & -R_m \end{bmatrix} \underline{x} \qquad \text{state variable} = \underline{x} = \begin{bmatrix} v_1 \\ i_4 \\ i_5 \end{bmatrix}$$

$$s\underline{1}_3 \underline{x} = \underline{E}^{-1}\underline{A}\,\underline{x} = A\underline{x} \qquad E = \begin{bmatrix} C & 0 & 0 \\ 0 & L_1 & 0 \\ 0 & 0 & L_2 \end{bmatrix}$$

$$x(t) = e^{At} x(0) \qquad E^{-1} = \begin{bmatrix} C^{-1} & 0 & 0 \\ 0 & 1/L_1 & 0 \\ 0 & 0 & 1/L_2 \end{bmatrix}$$

$$(sE - A) X(s) = X(0)$$

$$X(s) = (sE - A)^{-1} x(0)$$

$\det(sE - A) = 0$ & solve for s but instead find L_1, L_2, C, g_m to oscillate

Choose $s = j\omega_0$ as oscillation "complex" frequency.

Work with the degree three polynomial in s

$$p(s) = \det(sE - A) = c_3 s^3 + c_2 s^2 + c_1 s + c_0$$

$$\det\begin{bmatrix} sC & 1 & 0 \\ -R_m G_0 & sL_1 + R_m & R_m \\ 1 - R_m G_0 & R_m & sL_2 + R_m \end{bmatrix} = sC \det\begin{bmatrix} sL_1 + R_m & R_m \\ R_m & sL_2 + R_m \end{bmatrix} - 1 \det\begin{bmatrix} -R_m G_0 & R_m \\ 1 - R_m G_0 & sL_2 + R_m \end{bmatrix}$$

$$= sC\left[(sL_1 + R_m)(sL_2 + R_m) - R_m^2\right] - \left[-sL_2 R_m G_0 - \cancel{R_m^2 G_0} - R_m + \cancel{R_m^2 G_0}\right]$$

$$= L_1 L_2 C s^3 + (L_1 + L_2) R_m C s^2 + L_2 R_m G_0 s + R_m$$

= characteristic polynomial $p(s)$

$$P(j\omega_0) = -(L_1 + L_2) R_m C \omega_0^2 + R_m + j\left[-L_1 L_2 C \omega_0^3 + L_2 R_m G_0 \omega_0\right]$$

desire this = 0 for oscillator; $P(s) = C_0(s^2+\omega_0^2)(s+G_0)$

real = 0

$$-(L_1+L_2)R_m C\omega_0^2 = -R_m \Rightarrow \omega_0^2 = \frac{1}{(L_1+L_2)C} \Rightarrow \omega_0 = \frac{1}{\sqrt{(L_1+L_2)C}}$$

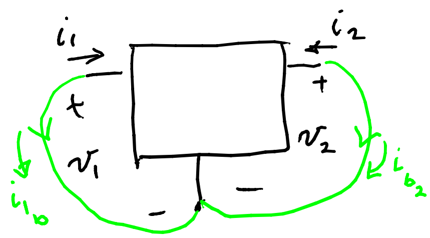
as $R_m \neq 0$

j term = 0

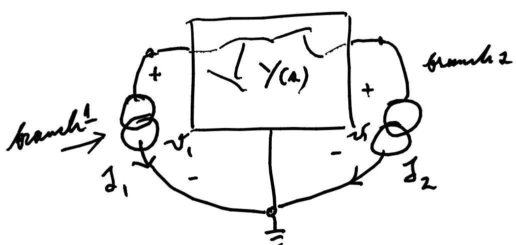
$$L_1L_2C\,\omega_0^2 = L_2R_mG_0 \quad \text{as } \omega_0 \neq 0$$

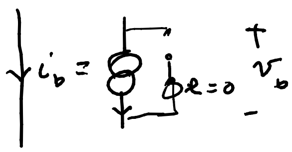
$$\frac{L_1L_2\cancel{C}}{(L_1+L_2)\cancel{C}} = L_2R_mG_0 \Rightarrow R_m = \frac{L_1}{G_0(L_1+L_2)} = \frac{1}{g_m+G_0}$$

$\therefore$ choose g_m to make it oscillate



$$Y(s)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$





$i_b = i + j = j$ for 1st 2 branches

$$v_b = \left.\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}\right\} v_t$$

assume here no internal node but branches

$$v_b = \begin{bmatrix} v_1 \\ v_2 \\ v_n \end{bmatrix}_b = \begin{bmatrix} v_1 \\ v_2 \\ v_n \end{bmatrix}$$