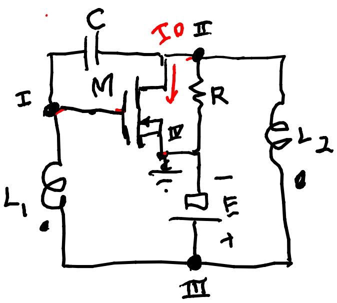
EE 610
09/02/10



$\Rightarrow$ graph

$V_{DS} = E = V_{GS}$

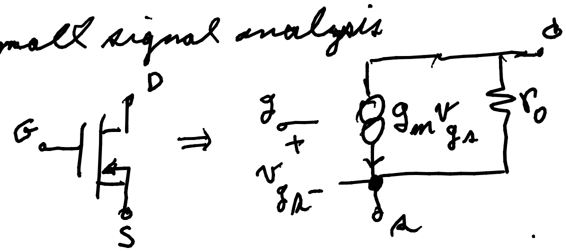
$V_{DS} > V_{GS} - V_{TH}$ > 0 for NMOS enhancement mode

if $E >> V_{TH}$

transistor is in saturation

drain current at bias $= I_D = \beta(V_{GS} - V_{TH})^2(1 + \gamma V_{DS})$

for small signal analysis



$i_D = \beta(v_{GS} - V_{TH})^2(1 + \gamma v_{DS})$

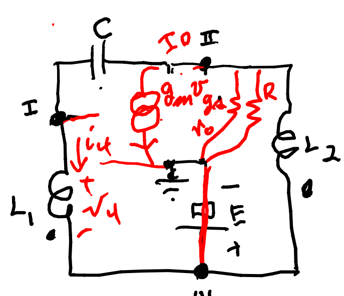
$$g_m = \frac{\partial i_D}{\partial v_{GS}}\Big|_{\cancel{bias}\ Q} = 2\beta(v_{GS} - V_{TH})(1 + v_{DS})\Big|$$

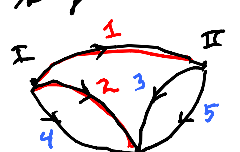
$Q;\ V_{GS} = E,\ V_{DS} = E$

$$g_m = 2\frac{I_D}{V_{GS} - V_{TH}};$$

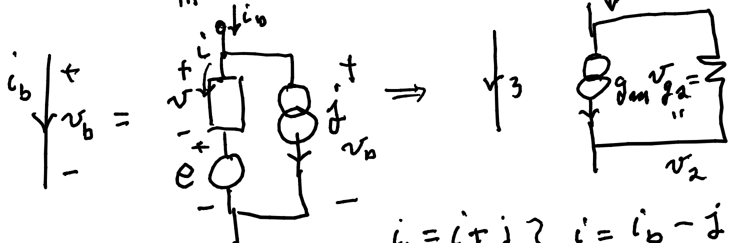
$$g_o = \frac{1}{r_o} = \frac{\partial i_D}{\partial v_{DS}}\Big|_Q = \beta(v_{GS} - V_{TH})^2\gamma\Big|_Q = \frac{I_D\,\gamma}{1 + \gamma V_{DS}}$$

small signal analysis





III = gnd



$$\frac{1}{g_o + G} = R_{eq}\quad v_3$$

$i_b = i + j$, $v_b = v + e$; $i = i_b - j$, $v = v_b - e$

assume have linearized around the operating (= Q = bias) point; $s = d/dt$

$$\begin{bmatrix} Cs & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & g_m & g_0+G & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & L_1 s & 0 \\ 0 & 0 & 0 & 0 & L_2 s \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

unconnected equations

$$\Rightarrow 0 = e\, i_b = \left[1_t \,\vdots\, K \right] \begin{bmatrix} i_t \\ i_\ell \end{bmatrix}_b \; ; \quad 0 = \mathcal{T} v_b = \left[-K^T \,\vdots\, 1_\ell \right] \begin{bmatrix} v_t \\ v_\ell \end{bmatrix}_b$$

$$\begin{bmatrix} v_t \\ v_\ell \end{bmatrix} = \begin{bmatrix} 1_t \\ \hdashline K^T \end{bmatrix} v_t \qquad \qquad 0 = -K^T v_t + v_\ell \quad \Rightarrow v_\ell = K^T v_t$$

$$\Rightarrow v_b = e^T v_t \,, \quad i_b = \mathcal{T}^T i_\ell$$

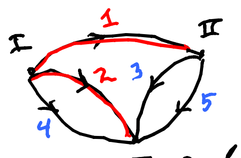
Note
$$\underbrace{v_b^T \cdot i_b}_{\substack{\text{power in our circuit}\\ \text{coming in from}\\ \text{the outside}\\ \equiv 0}} = (e^T v_t)^T \cdot \mathcal{T}^T i_\ell = \underbrace{v_t^T}_{\text{a row } t \text{ vector}} \underbrace{[e \cdot \mathcal{T}^T]}_{\text{a } t \times \ell \text{ matrix}} \underbrace{i_\ell}_{\text{a column } \ell\text{-vector}}$$

$$e \cdot \mathcal{T}^T = \left[1_t \,\vdots\, K \right] \begin{bmatrix} -K \\ \hdashline 1_\ell \end{bmatrix} = -K + K = 0_{t \times \ell}$$

$$v = v_b - e \,, \quad i = i_b - j$$
$$\;\; = e^T v_t - e \qquad = \mathcal{T}^T i_\ell - j$$

$$A(s) v = B(s) i$$
$$A(s) e^T v_t - A(s) e = B(s) \mathcal{T}^T i_\ell - B(s) j$$
$$A(s) e^T v_t - B(s) \mathcal{T}^T i_\ell = A(s) e - B(s) j$$

b-equation $\left\{ \left[A(s) e^T \,\vdots\, -B(s) \mathcal{T}^T \right] \underbrace{\begin{bmatrix} v_t \\ i_\ell \end{bmatrix}}_{\text{b-vector}} = A(s) e - B(s) j \right.$



$$e = \begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \; ; \; e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \; j = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = v_b = e^T v_t$$
$$i = i_b = \sigma^T i_\ell$$

$$\underbrace{\begin{bmatrix} Cs & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & g_m & g_0+G & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{A(s)} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}}_{e^T} \underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{v_t}$$

$$g_0 + G = G_0$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & L_1 s & 0 \\ 0 & 0 & 0 & 0 & L_2 s \end{bmatrix}}_{B(s)} \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\sigma^T} \underbrace{\begin{bmatrix} i_3 \\ i_4 \\ i_5 \end{bmatrix}}_{i_\ell}$$

$$x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$$

$$\begin{bmatrix} Cs & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ -G_0 & g_m+G_0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -L_1 s & 0 \\ -1 & 1 & 0 & 0 & -L_2 s \end{bmatrix} x = 0_5$$

$$s \begin{bmatrix} C & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_1 & 0 \\ 0 & 0 & 0 & 0 & -L_2 \end{bmatrix} x = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 \\ G_0 & -(g_m+G_0) & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix} x$$

$$E \frac{dx}{dt} = Ax$$

here E is singular so close to state variable equation.

$$\frac{dx}{dt} = Ax$$

take unilateral Laplace transform

$$sE \, \mathcal{L}[x] = A \, \mathcal{L}[x] + E x_0$$

↑ initial condition

$$\mathcal{L}[x] = [sE - A]^{-1} E x_0$$