

1. [S matrix for quantum computers]

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The above two (frequency independent) matrices are the key ones for quantum computing where H is for the Hadamard transformation and C is for the controlled not.

- Show that these can be considered as scattering matrices of passive circuits.
 - Treat each as a scattering matrix and realize by a passive circuit.
- { Although not necessary for working this problem it is of interest to know that they transform qubits where H is for one qubit and C is for the tensor product of two qubits. Scattering vectors can here be considered as qubits if they have norm 1).

2. [even part zeros]

$$\text{For } y(s) = \frac{s^2 + bs + c}{s^2 + s + 1}$$

- Show that this is pr for b & c both positive.
- What if one or both are zero?
- Assuming b & c both positive find the even part zeros of y(s) and determine for which values these zeros are real.

3. [S matrix normalizations]

Two engineers measure the scattering matrix of a 2-port, one obtains $S_1(s)$ with respect to the terminations having $r_1 = z_0$ and the other obtains $S_2(s)$ with respect to $r_2 = z_0$. In order to tell if their measurements agree it is necessary to relate S_2 to S_1 . Thus, give S_2 as a function of S_1 and r_1 & r_2 and check that if $r_1 = r_2$ then $S_2 = S_1$ (hint, the current versus voltage laws are the same in each instance, that is, if Y exists it would be the same for both measurements).