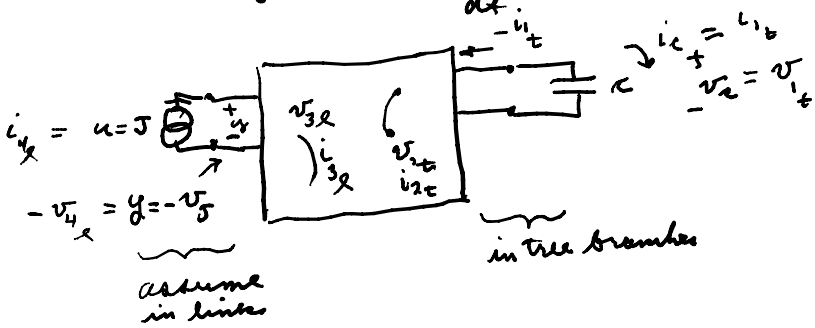
EE 610
10/27/09

Nonlinear passivity

$u(t)$ = input (currents)
$y(t)$ = output (voltages in where currents are applied)
$y^T(t)u(t)$ = power in
$\mathcal{E}(t)$ = storage function (energy in capacitors)

The system with this y, u pair is passive if

$$y^T(t)u(t) \geq \frac{d\mathcal{E}(t)}{dt} \quad \text{for all } t$$



$$v_b = \begin{bmatrix} v_t \\ \hline v_\ell \end{bmatrix} = \begin{bmatrix} v_c \\ v_{2_t} \\ \hline v_{3_\ell} \\ v_J \end{bmatrix} \qquad i_b = \begin{bmatrix} i_t \\ \hline i_\ell \end{bmatrix} = \begin{bmatrix} i_{1_t} \\ i_{2_t} \\ \hline i_{3_\ell} \\ J \end{bmatrix}$$

$$\mathcal{E}(t) = \int_0^t v_c^T(\tau)\, i_c(\tau)\, d\tau \; ; \quad \dot{\mathcal{E}}(t) = v_c^T(t) \cdot i_c(t)$$

$$-y = \begin{bmatrix} v_c \\ v_{2_t} \\ \hline v_{3_\ell} \\ v_J \end{bmatrix} = v_b = \left[\begin{array}{c|c} 1_t & \\ \hline k_{11} & k_{12} \\ k_{21} & k_{22} \end{array}\right] \begin{bmatrix} v_c \\ v_{2_t} \end{bmatrix} = C^T v_t \qquad \begin{bmatrix} i_c \\ i_{2_t} \\ \hline i_{3_\ell} \\ J \end{bmatrix} = i_b = \mathcal{J}^T i_\ell = \left[\begin{array}{cc} -k_{11}^T & -k_{21}^T \\ -k_{12}^T & -k_{22}^T \\ \hline \multicolumn{2}{c}{1_\ell} \end{array}\right] \begin{bmatrix} i_{3_\ell} \\ J \end{bmatrix}$$

from 4th "row" of v_b: $\quad y = -k_{21} v_c - k_{22} v_{2_t} \; ; \quad y^T u = -\left(v_c^T k_{21}^T + v_{2_t}^T k_{22}^T\right) J$

from 2nd "row" of i_b: $\quad i_{2_t} = -k_{12}^T i_{3_\ell} - k_{22}^T J \implies -k_{22}^T J = i_{2_t} + k_{12}^T i_{3_\ell}$

into $y^T u$: $\quad y^T u = -v_c^T k_{21}^T J + v_{2_t}^T \left[i_{2_t} + k_{12}^T i_{3_\ell} \right]$

$$= -v_c^T k_{21}^T J + v_{2_t}^T k_{12}^T i_{3_\ell} + \underbrace{v_{2_t}^T i_{2_t}}_{\text{power into devices inside in the trees}}$$

from 1st "row" of i_b: $\quad i_c = -k_{11}^T i_{3_\ell} - k_{21}^T J$

$$\implies -k_{21}^T J = i_c + k_{11}^T i_{3_\ell}$$

into $y^T u$: $\quad y^T u = v_c^T \left[i_c + k_{11}^T i_{3_\ell} \right] + v_{2_t}^T k_{12}^T i_{3_\ell} + v_{2_t}^T i_{2_t}$

$$= \dot{\mathcal{E}}_c + \left[v_c^T k_{11}^T + v_{2_t}^T k_{12}^T \right] i_{3_\ell} + v_{2_t}^T i_{2_t}$$

from 3rd "row" of v_b: $\quad v_{3_\ell} = k_{11} v_c + k_{12} v_{2_t} \implies v_{3_\ell}^T = v_c^T k_{11}^T + v_{2_t}^T k_{12}^T$

into $y^T u$:

$$\underset{\text{power in}}{p(t)} = y^T u = \dot{\mathcal{E}}_\ell + \underbrace{v_{3_\ell}^T i_{3_\ell} + v_{2_t}^T i_{2_t}}_{\substack{\text{power into coupling} \\ \text{circuit branches (which can be} \\ \text{nonlinear)}}}$$

if passive these should be non-negative

shows $y^T u - \dot{\mathcal{E}}_\ell \geq 0$ (for all t) definition of passivity used in nonlinear control

$\therefore$ desire to set up semistate equations to handle nonlinear devices.

For capacitors $i_\ell = C\dot{v}_\ell$ assume they are in the tree

For input currents, J; assume in links.

$$i_{2_t} = g_{2_t}(v_b) = g_{2_t}(e^T v_t) \qquad x = \begin{bmatrix} v_\ell \\ v_{2_t} \\ i_{3_\ell} \\ J \end{bmatrix}$$

v_{2_t} ← for tree branches not with capacitor

i_{3_ℓ}, J ← for links branches not current sources

$$i_{3_\ell} = g_{3_t}(v_b) = g_{3_t}(e^T v_t)$$

$$i_b = \mathcal{J}^T i_\ell + J_{\text{sources}} \; ; \quad v_b = e^T v_t$$

$$= \begin{bmatrix} i_\ell \\ i_{2_t} \\ i_{3_b} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ J \end{bmatrix} \qquad = \left[\begin{array}{c} \mathcal{L}_t \\ \hline \begin{matrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{matrix} \end{array}\right] \begin{bmatrix} v_\ell \\ v_{2_t} \end{bmatrix}$$

$$= \begin{bmatrix} -k_{11}^T & -k_{21}^T \\ -k_{12}^T & -k_{22}^T \\ \mathbb{1}_\ell & \end{bmatrix} \begin{bmatrix} i_{3_\ell} \\ J \end{bmatrix}$$

$$i_\ell = C\dot{v}_\ell = \underbrace{\begin{bmatrix} C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{v}_\ell \\ \dot{v}_{2_t} \\ \dot{i}_{3_\ell} \\ \dot{J} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -k_{11}^T & -k_{21}^T \\ 0 & 0 & +k_{12}^T & +k_{22}^T \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ g_{2_t}(e^T v_t) \\ g_{3_\ell}(e^T v_t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbb{1}_\ell \end{bmatrix} u}_{\substack{\text{nonlinear dynamics in} \\ \text{semistate form}}}$$

$$E\dot{x} = \mathcal{A}(x, t) + Bu \; ; \quad u = [0 \; 0 \; 0 \; J^T]^T$$

$y = Cx$ ← after specify an output in terms of voltages & currents in the circuit (all functions of tree voltage & links currents)

If have nonlinear capacitors (or inductors) need to transform via nonlinear non-dynamic circuits loaded by linear capacitors. Also can do for time-varying. So get in general

$$E\dot{x} = \mathcal{A}(x,t) + Bu$$
$$y = Cx$$