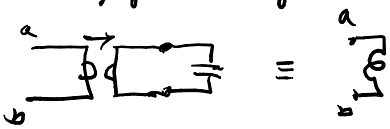
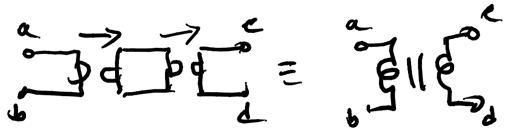
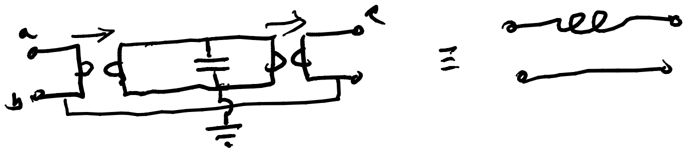
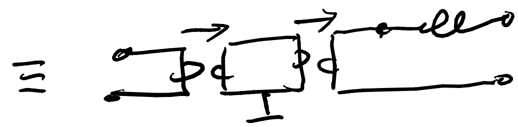
some gyrator equivalences

EE 610
10/22/09









Richards' function is PR for k real > 0

$$y_1(s) = y(k)\cdot \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$$

$$= S_1(s) = \frac{1 - y_1(s)/y(k)}{1 + y_1(s)/y(k)} \quad \text{should be BR if } y(s) \text{ is PR}$$

$$= S_{y_1}(s)$$

$$= \frac{k\,y(s) - s\,y(k) - k\,y(k) + s\,y(s)}{k\,y(s) - s\,y(k) + k\,y(k) - s\,y(s)}$$

$$= \frac{s(y(s) - y(k)) + k(y(s) - y(k))}{-s(y(s) + y(k)) + k(y(s) + y(k))}$$

$$= \frac{(s+k)(y(s) - y(k))}{(-s+k)(y(s) + y(k))} = \frac{s+k}{s-k}\cdot\frac{y(k) - y(s)}{y(k) + y(s)}$$

$$= \frac{s+k}{s-k}\cdot\frac{1 - y(s)/y(k)}{1 + y(s)/y(k)} = \left(\frac{s+k}{s-k}\right)\cdot \underbrace{S_y(s)}_{\text{BR}}$$

$$\left.\frac{s+k}{s-k}\right|_{s=j\omega} = \frac{k+j\omega}{-k+j\omega} = (-1)\left(\frac{k+j\omega}{k-j\omega}\right)$$

$$\left|\frac{s+k}{s-k}\right|_{s=j\omega} = 1 = |-1|\,\frac{|k+j\omega|}{|k-j\omega|} = 1\cdot\frac{\sqrt{k^2+\omega^2}}{\sqrt{k^2+\omega^2}} = 1$$

$$1-|S_{y_L}(j\omega)|^2 = 1-|S_y(j\omega)|^2 \geq 0 \quad \text{as } y \text{ is assumed PR}$$

$\Rightarrow$ Richards' function for a PR $y(s)$ is PR

$$y_L(s) = y(k)\cdot\frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$$

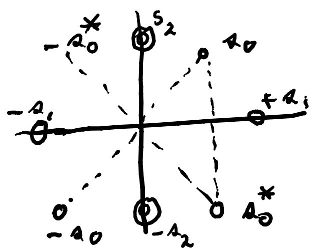
$s+k$ cancels

$\delta[y_L] \leq \delta[y]$

if $y(k) = -y(-k)$ then also $s-k$ cancels

$$\mathrm{Ev}\,y(s)\Big|_{s=k} = \frac{y(k)+y(-k)}{2} = 0$$

as $k\,y(k) - (-k)\,y(-k)$
$= k\,y(k) + k\,y(-k)$
$= k\,y(k) + k[-y(k)] = 0$

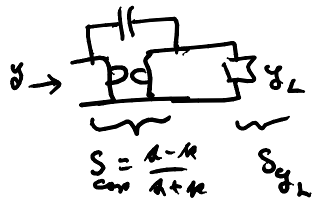


zeros of even part

if s_0 is a zero of the even part then $-s_0$ is also a zero of the even $y(s)+y(-s)$

$y(s_0)+y(-s_0)=0$

$$S_y(s) = \underbrace{\left(\frac{s-k}{s+k}\right)}_{BR}\underbrace{S_{y_L}}_{BR}$$



$$2\,\mathrm{Ev}\,\frac{y_L(s)}{y(k)} = \frac{y_L(s)}{y(k)} + \frac{y_L(-s)}{y(k)} = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)} + \frac{k\,y(k) + s\,y(-s)}{k\,y(-s) + s\,y(k)}$$

$$= \frac{(k\,y(k) - s\,y(s))(k\,y(-s) + s\,y(k)) + (k\,y(k) + s\,y(-s))(k\,y(s) - s\,y(k))}{D(s)\,D(-s)}$$

$$= \Big(k^2 y(k)y(-s) + ks\,\cancel{y(k)^2} - s\cancel{k\,y(s)y(-s)} - s^2 y(k)y(s)$$
$$k^2 y(k)y(s) - ks\,\cancel{y(k)^2} + sk\,\cancel{y(s)y(-s)} - s^2 y(k)y(-s)\Big)\Big/ DD_*$$

$$= (k^2-s^2)\,y(k)y(-s) + (k^2-s^2)(y(k)y(s))\,/\,DD_*$$

$$= (k^2-s^2)\,y(k)\underbrace{[y(s)+y(-s)]}_{\text{still has other zeros of } \mathrm{Ev}\,y(s)}$$

Example: If $y(s)$ is lossless (PR)

then $y(s)+y(-s)\equiv 0$ $\therefore$ all real $k>0$ are zeros of $Ev\,y(s)$

gives $\delta[y_L]=\delta[y]-1$

$$y(s)=\frac{s(s^2+9)}{(s^2+4)} \qquad \text{choose } k=2\,; \quad y(s)\Big|_{s=2}=\frac{2(13)}{8}>0$$

$$y(2)+y(-2)=\frac{2\times 13}{8}-\frac{2\times 13}{8}=0$$

$y(2)=13/4$

$$y_L(s)=\frac{13}{4}\left[\frac{k\,y(k)-s\,y(s)}{k\,y(s)-s\,y(k)}\right]=\frac{13}{4}\left[\frac{2\times\frac{13}{4}-\frac{s^2(s^2+9)}{s^2+4}}{2\,\frac{s(s^2+9)}{s^2+4}-\frac{13}{4}s}\right]$$

$$\frac{y_L(s)}{13/4}=\frac{\frac{13}{2}(s^2+4)-s^4-9s^2}{2s^3+18s-\frac{13}{4}s^3-13s}=\frac{1}{s}\left[\frac{-s^4-(9-\frac{13}{2})s^2+26}{(2-\frac{13}{4})s^2+(18-13)}\right]$$

here $(s+2)(s-2)$ should factor num. & den

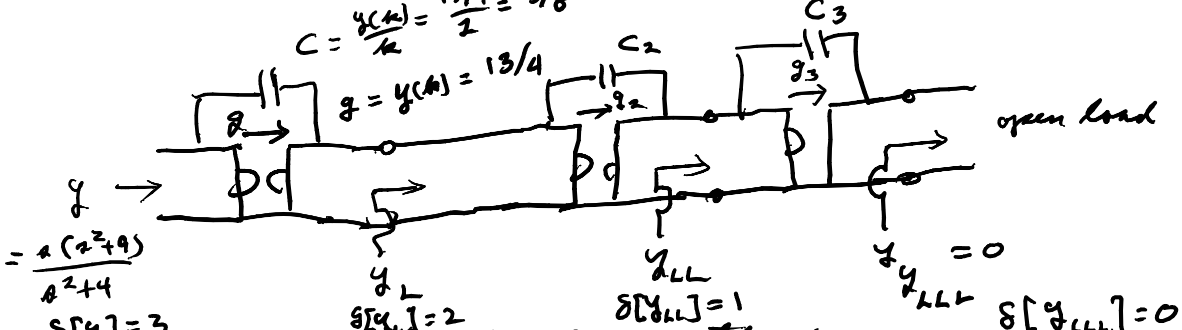
$=s^2-4$

$$\begin{array}{r|l} & -s^2-13/2 \\ \hline s^2-4 & -s^4-\frac{5}{2}s^2+26 \\ & -s^4+4s^2 \\ \hline & -\frac{13}{2}\quad +\frac{13\times4}{2} \\ & \qquad 0 \end{array} \qquad \begin{array}{r|l} & -5/4 \\ \hline s^2-4 & -\frac{5}{4}s^2+5 \end{array}$$

$$y_L(s)=\frac{13}{4}\cdot\frac{1}{s}\,\frac{-s^2-13/2}{-5/4}=\frac{13}{5}\cdot\frac{s^2+13/2}{s} \quad \text{this PR},\ \delta[y_L]=2$$

degree $=\delta[y]=3$

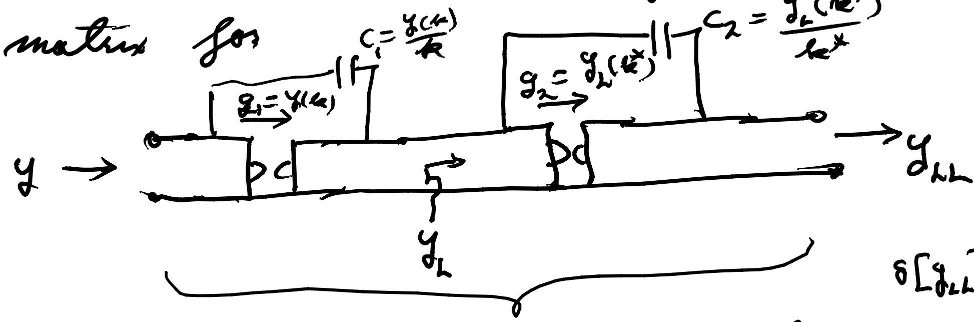
$C=\dfrac{y(k)}{k}=\dfrac{13/4}{2}=13/8$ $\quad g=y(k)=13/4$



gives another canonical synthesis.

Can also use complex zeros k of the even part

gives complex C, g for $y_L(s)$; on $y_L(s)$ use k^* for the "k" in Richards' function, then y_{LL} will be PR & the two complex capacitors & gyrators will combine in the coupling admittance matrix for



$\delta[y_{LL}] = \delta[y] - 2$

Y_{comp} that is PR & lossless

shows can synthesize a PR y using only 1 resistor & $\delta[y]$ capacitors (& gyrators)