EE 610
10/20/09

$$R_y(s) = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$$

lot: $y(s), z(s) \rightarrow$ PR = positive real, rational
$S(s) \longrightarrow$ BR = bounded real, rational

all are analytic in $\sigma > 0$, no poles
y & z $\rightarrow$ no zeros in $\sigma > 0$ as $\frac{1}{y} = z$, $\frac{1}{y}$ or $\frac{1}{z}$ is PR
$S(s)$ can have zeros in $\sigma > 0$ ($\Rightarrow \frac{1}{S}$ is not BR)
(example: $y(s) = s$ is PR, $S(s) = \frac{1-y}{1+y} = \frac{1-s}{1+s}$ has a zero at $s = 1$

if PR & lossless then $y(s) = -y(-s)$ — all poles & zeros are on $s = j\omega$, alternate, simple & have positive residue
here $Ev\, y(s) = 0$

if BR & lossless then $S(s)\,S^T(-s) = 1_n$ if $n = 1$, $S(-s) = \frac{1}{S(s)}$

$$\left.\begin{aligned} Even\ a(s) &= \frac{a(s) + a(-s)}{2} \\ od\ a(s) &= \frac{a(s) - a(-s)}{2} \end{aligned}\right\} \text{ add } a(s) = Ev\,a(s) + Od\,a(s)$$

$$\left.\begin{aligned} Re\ a(s) &= \frac{a(s) + a^*(s)}{2} \\ j\,Im\ a(s) &= \frac{a(s) - a^*(s)}{2} \end{aligned}\right\} \text{ add } a(s) = Re\,a(s) + j\,Im\,a(s)$$

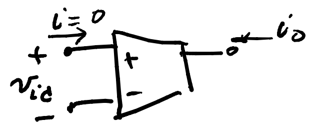
for $s = j\omega$ if rational with real coefficients
$$a^*(j\omega) = a(-j\omega) \Rightarrow Re\,a(j\omega) = Ev\,a(j\omega)$$
$$od\,a(j\omega) = j\,Od\,a(j\omega)$$

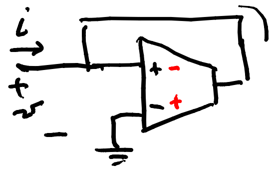
PR lossless: $y(s) = -y(-s) \Rightarrow Ev\,y(s) = 0 = \frac{y(s) + y(-s)}{2}$
$$Ev\,y(j\omega) = Re\,y(j\omega) = 0 \Rightarrow y(j\omega) = j\,Im\,y(j\omega)$$
$$\Rightarrow y(s) = \text{odd}$$

For lossless $Ev\,y(s) = 0$

OTA-C synthesis: OTA = operational transconductance amplifier
= VCCS (op-amp is a VCVS)
v_{out} = indeterminate
$i_{out} = g_m(v_{in})$, $v_{in} = v_+ - v_-$

$$\begin{bmatrix} i_o \end{bmatrix} = \begin{bmatrix} g_m \end{bmatrix}\begin{bmatrix} v_{id} \end{bmatrix}$$





$i = -g_m v \Rightarrow$

$R = \frac{1}{-g_m}$

$$E\dot{x} = Ax + Bu$$
$$y = Cx$$

assume $u = v$, $y = i$

find a P&Q, nonsingular & transform the equation to make E diagonal: $x = Q\hat{x}$; P&Q constant

$$PEQ\dot{\hat{x}} = PAQ\hat{x} + PBu$$

output = $y = CQ\hat{x}$

$$Y(s) = CQ(sPEQ - PAQ)^{-1}PB \cdot U(s)$$

$$T(s) = CQ(sPEQ - PAQ)^{-1}PB$$
$$= CQ(P(sE - A)Q)^{-1}PB$$
$$= CQQ^{-1}(sE - A)^{-1}P^{-1}PB$$
$$= C(sE - A)^{-1}B$$

choose P&Q to bring $\hat{E} = \begin{bmatrix} 1_r & 0 \\ 0 & 0 \end{bmatrix} = PEQ$, r = rank of E

This gives $\hat{A} = PAQ$, $\hat{B} = PB$, $\hat{C} = CQ$

$$\hat{E}\dot{\hat{x}} = \begin{bmatrix} 1_r & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} \hat{A} \end{bmatrix}\hat{x} + \hat{B}u$$

, $\hat{x}_1$ = r-vector

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \begin{array}{l} \} r \text{ rows} \\ \} k-r \text{ rows} \end{array}$$

$$y = \hat{C}\hat{x}$$

if x were a k-vector

current into r capacitors & $k-r$ opens

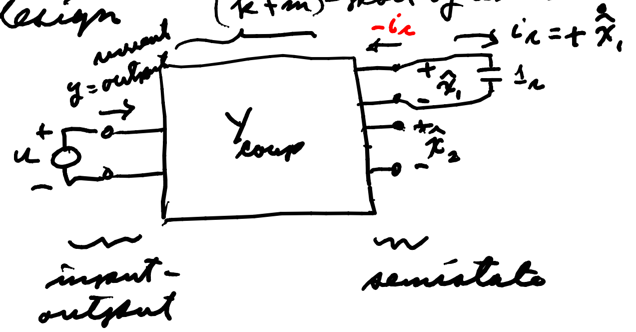
$$\begin{bmatrix} -1_r\dot{\hat{x}}_1 \\ 0 \\ y \end{bmatrix} = \begin{bmatrix} -E\dot{\hat{x}} \\ y \end{bmatrix} = \begin{bmatrix} -\hat{A} & -\hat{B} \\ \hat{C} & 0 \end{bmatrix}\begin{bmatrix} \hat{x} \\ u \end{bmatrix} \begin{array}{l} \} \text{voltages} \\ \} \text{input voltage} \} m\text{-vector} \end{array}$$

output currents m-vector

an admittance matrix

$$Y_{comp.} = \begin{array}{c} \begin{bmatrix} -\hat{A} & -\hat{B} \\ \hat{C} & 0 \end{bmatrix} \begin{array}{l} \} k \\ \} m \end{array} \\ \underbrace{\quad}_{k}\ \underbrace{\quad}_{m} \end{array}$$

To design (k+m)-port of admittance Y_{comp}



Ex: assume this is given

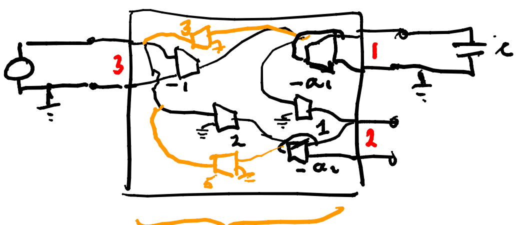
$$\begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ -1 & a_2 \end{bmatrix} x + \begin{bmatrix} 1 \\ -2 \end{bmatrix} v_i$$

$$i_o = \begin{bmatrix} 3 & -5 \end{bmatrix} x$$

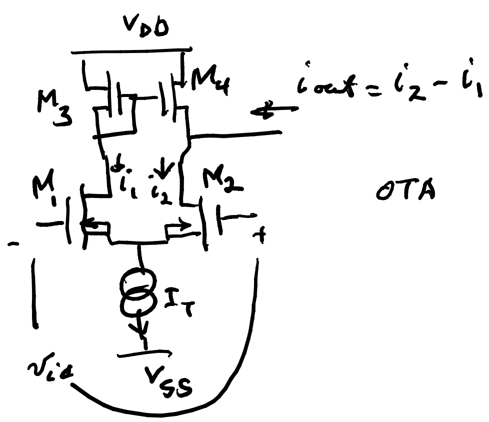
$a_1 < 0$

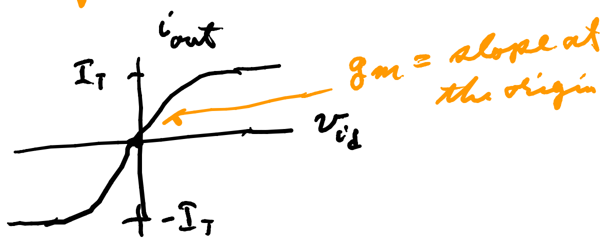
$a_2 > 0$, $c > 0$

$$Y_{comp} = \begin{bmatrix} -a_1 & 0 & -1 \\ 1 & -a_2 & 2 \\ 3 & -5 & 0 \end{bmatrix}$$



OTA realization of Y_{comp}





Back to Richards':

$$R_y(s) = \frac{k\, y(k) - s\, y(s)}{k\, y(s) - s\, y(k)}$$

if $y(k) = -y(-k)$ then

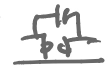
$k\, y(k) - (-k)\, y(-k)$ @ $s = -k$

$= k\, y(k) - k\, y(k) = 0$ of num

also $s = -k$ is a zero of den.

then $s-(-k)$ cancels , already $s-k$ cancelled

& the degree $\delta[R_y(s)] = \delta[y(s)] - 1$

$\therefore$ if have real zeros of the even part of $y(s)$ can repeat  extraction until the $\delta[y_L] = 0$

$\therefore$ if y_L is lossless when y is lossless this gives a 5th synthesis of lossless $y(s)$