EE 610
10/15/09

Richards' function, p. 361
Lossless 1-port synthesis, pp. 342-

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$$y(s) = \frac{s(s^2+6)}{(s^2+4)(s^2+9)} = \frac{s^3+6s}{s^4+13s^2+36}$$

For 2nd Cauer; continued fraction expansion about $s=0$

$$\frac{1}{y(s)} = z(s) = \frac{36+13s^2+s^4}{6s+s^3} \Rightarrow y(s) = \frac{6s+s^3}{36+13s^2+s^4} = \frac{1}{\frac{6}{s} + \frac{7s^2+s^4}{6s+s^3}}$$

$$= \frac{1}{\frac{6}{s} + \frac{1}{\left\{\frac{6s+s^3}{7s^2+s^4}\right\}}}$$

$$\frac{\cancel{s}(6+s^2)}{s^{\cancel{2}}(7+s^2}$$

$$6s+s^3 \overline{\smash{)}\,36+13s^2+s^4} \quad \frac{6}{s}$$

$$36+6s^2$$

$$7s^2+s^4 \overline{\smash{)}\,6s+s^3} \quad \frac{6}{7s}$$

$$6s+\frac{6}{7}s^3$$

$$\frac{1}{7}s^3 \overline{\smash{)}\,7s^2+s^4} \quad \frac{49}{s}$$

$$7s^2$$

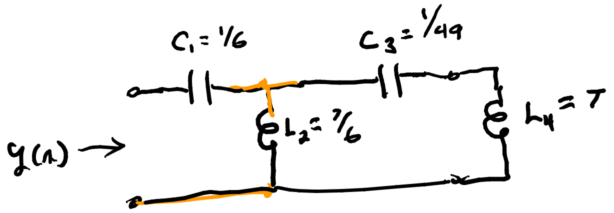
$$s^4 \overline{\smash{)}\,\frac{1}{7}s^3} \quad \frac{1}{7s}$$

$$\frac{1}{7}s^3$$

$$0$$

$$y(s) = \frac{1}{\underset{z_1(s)=}{\frac{6}{s}} + \frac{1}{\underset{y_2(s)=}{\frac{6}{7s}} + \frac{1}{\underset{z_3(s)=}{\frac{49}{s}} + \frac{1}{\underset{y_4=}{\frac{1}{7s}}}}}} = \frac{6s+s^3}{36+13s^2+s^4}$$

= continued fraction expansion about $s=0$



2nd Cauer synthesis of
$$y(s) = \frac{s^3+6s}{s^4+13s^2+36}$$

$\therefore$ have 4 syntheses of this $y(s)$, each of which uses the minimum number, the degree of $y(s)$, in each

$\delta[y(s)]$ = degree (here = highest power of s seen in $y(s)$)

[each step reduced the degree by 1, so synthesis of a finite lossless $y(s)$ by 1st or 2nd Foster or Cauer]

$$y(s) = -y(-s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} \qquad k_i \geq 0$$

For $s = j\omega$; $y(j\omega) = j\,B(\omega)$; $\frac{dB(\omega)}{d\omega}$

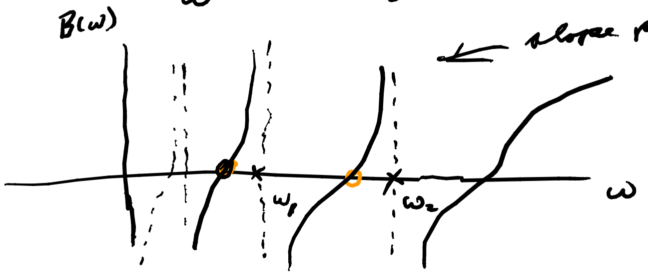
$$B(\omega) = \frac{y(j\omega)}{j} = -\frac{k_0}{\omega} + k_\infty \omega + \sum_{i=1}^{n} \frac{2k_i\omega}{-\omega^2+\omega_i^2}$$

$$\frac{dB(\omega)}{d\omega} = \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{n} \left\{ \frac{2k_i}{-\omega^2+\omega_i^2} - \frac{2k_i\omega(-2\omega)}{(-\omega^2+\omega_i^2)^2} \right\}$$

$$\| $$

$$\frac{2k_i}{(-\omega^2+\omega_i^2)^2}\{\underbrace{-\omega^2+\omega_i^2+2\omega^2}_{\omega^2+\omega_i^2}\}$$

$$= \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{n} \frac{2k_i(\omega^2+\omega_i^2)}{(-\omega^2+\omega_i^2)^2} \geq 0$$



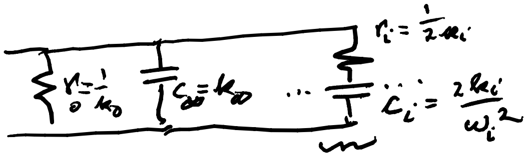
← slope positive

shows between every pole there is a zero (allows us to easily create reactance functions)

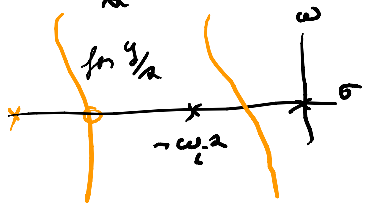
Look at RC PR functions ⇒ had $y_{RC}(s)$ given one can replace R's by L's ⇒ LC circuit Synthesize the LC circuit by its $y_{LC}(s)$ then replace every L by an R gives y_{RC}

$$y_{LC} = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} = \underset{\substack{\uparrow \\ L = 1/k_0}}{\frac{k_0}{s}} + k_\infty s + \sum_{i=1}^{n} \frac{1}{\underset{\substack{\uparrow \\ L = \frac{1}{2k_i}}}{\frac{s}{2k_i}} + \frac{\omega_i^2}{2k_i s}}$$

$$y_{RC} = k_0 + k_\infty s + \sum_{i=1}^{n} \frac{1}{\frac{1}{2k_i} + \frac{\omega_i^2}{2k_i s}} = k_0 + k_\infty s + \sum_{i=1}^{n} \frac{2k_i s}{s+\omega_i^2}$$



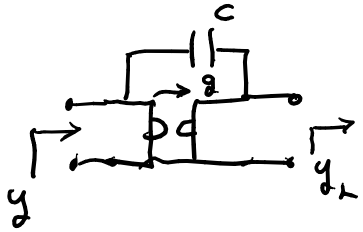
$$\frac{y_{RC}(s)}{s} = \frac{k_0}{s} + k_\infty + \sum_{i=1}^{n} \frac{2k_i}{s+\omega_i^2}$$ is now a partial fraction expansion



$$\Rightarrow \frac{d\,y(\sigma)/\sigma}{d\sigma} = -\frac{k_0}{\sigma^2} + \sum \frac{-2k_i}{(\sigma+\omega_i^2)^2} \leq 0$$

poles and zeros $\overset{\text{of } y/s}{\wedge}$ will alternate & are on the negative σ axis

Synthesis via the Richards' function



$$Y_{comp} = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix};$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$-y_L \cdot v_2 = i_2$$

$$(-sC - y_L)v_2 = (-sC-g)v_1 \Rightarrow v_2 = \frac{sC+g}{sC+y_L}$$

$$i_1 = y \cdot v_1 = \left[sC + (-sC+g)\left(\frac{sC+g}{sC+y_L}\right)\right] \cdot v_1$$

$$y = \frac{s^2C^2 + sCy_L - s^2C^2 + g^2}{sC+y_L} = \frac{sCy_L + g^2}{sC+y_L} = y$$

$$\Rightarrow sCy_L + g^2 = sCy + y_L y \Longrightarrow (sC - y)y_L = sCy - g^2$$

$$y_L = \frac{sCy - g^2}{sC - y} = g^2\left(\frac{1 - \frac{sC}{g} \cdot \frac{y(s)}{g}}{y - sC}\right) = \boxed{g\left(\frac{1 - \frac{sC}{g}\frac{y(s)}{g}}{\frac{y(s)}{g} - \frac{sC}{g}}\right)}$$

$$= g\left(\frac{\frac{sC}{g}\frac{y(s)}{g} - 1}{\frac{sC}{g} - \frac{y(s)}{g}}\right)$$

The Richards function:

p. 361

$$R(s) = \frac{k\,y(s) - s\,y(k)}{k\,y(k) - s\,y(s)}$$

use

$$R_y(s) = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$$

these are PR if k is real & >0 & $y(s)$ is PR

↖ R has a pole at $s=k$ & a zero & they cancel

I want y_L to look like $R_y(s)$

$$R_y(s) = \frac{k\,y(k)\left[1 - \frac{s}{k} \cdot \frac{y(s)}{y(k)}\right]}{k\,y(k)\left[\frac{y(s)}{y(k)} - \frac{s}{k}\right]} \sim g\left[\frac{1 - \frac{s}{(g/C)} \cdot \frac{y(s)}{g}}{\frac{y(s)}{g} - \frac{s}{(g/C)}}\right] = y_L$$

form $\frac{R_y(s)}{g}$ & $g = y(k)$, $k = g/C$ $\qquad g = y(k)$, $C = \frac{g}{k} = \frac{y(k)}{k}$

Choose k so that it is a zero of the even part of $y(s)$
for lossless any k is such a zero.