EE 610
10/06/09

Semistate equations:

$$E\dot{x} = a(x,t) + Bu$$
$$y = Cx$$

u = input = $u(t)$, an n-vector
y = output = $y(t)$, an m-vector
$x(t)$ = semistate, a k-vector

$\dot{x} = \dfrac{dx}{dt}$; E may be singular (as we have resistors OTA's, etc.)

E, B, C are constant matrices
E is $k \times k$, C is $m \times k$
B is $k \times n$

if linear & time-invariant
A is constant, $k \times k$

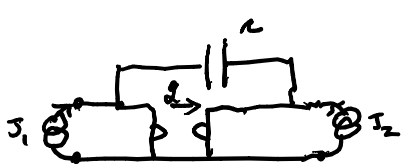
x can be $x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$ ← tree ← links, in which case $k = t + \ell = b$ = # of branches

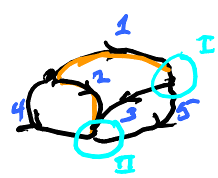
Laplace transforming $\mathcal{L}[\cdot]$
$$Es\,\mathcal{L}[x] = A\,\mathcal{L}[x] + B\,\mathcal{L}[u]$$
$$\mathcal{L}[y] = C\,\mathcal{L}[x]$$

$(Es - A)\,\mathcal{L}[x] = B\,\mathcal{L}[u]$, if $(Es-A)^{-1}$ exists then
$$\mathcal{L}[x] = (Es-A)^{-1}B\,\mathcal{L}[u]$$
$$\rightarrow \quad \mathcal{L}[y] = C(Es-A)^{-1}B \cdot \mathcal{L}[u] = T(s)\cdot\mathcal{L}[u]$$
$$T(s) = C(Es-A)^{-1}B = \underset{m\times n}{\text{transfer function}}\ \text{matrix}$$

If E is the identity, $E = 1_k$, then the semistate equations are state variable equations
(note if $E = 1_k$, $T(\infty) = 0_{m \times n}$ & resistors by themselves can not be directly handled)

Ex:





KCL: $\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b = \mathcal{E}\, i_b$

KVL: $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}_b = \mathcal{J}\, v_b$

$u = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}$, $y = \begin{bmatrix} v_4 \\ v_5 \end{bmatrix}_b$

$c\dot{v}_1 = i_1$

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_3 \\ i_4 \\ i_5 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$i_{4_b} = 0 + (-J_1)$
$i_{5_b} = 0 + (-J_2)$

$v_b = \mathcal{C}^T v_t$, $i_b = \mathcal{J}^T i_\ell$; $\quad v_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} v_t \quad i_b = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_3 \\ i_4 \\ i_5 \end{bmatrix}_b$

$$c\dot{v}_1 = c\dot{v}_{t_1} = i_{1_b} = i_{3_b} + i_{5_b}$$

$$\begin{bmatrix} c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dot{\begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}}_b = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ -g & g & 1 & 1 & 1 \\ 0 & -g & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}$$

$$\Rightarrow E\dot{x} = Ax + Bu$$

for the gyrator:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -g & g & 1 & 1 & 1 \\ 0 & -g & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} ;$$

For sources

$i_{4_b} = -J_1 \Rightarrow 0 = -1 \cdot i_{4_b} - J_1$

$i_{5_b} = -J_2 \Rightarrow 0 = -i_{5_b} - J_2$

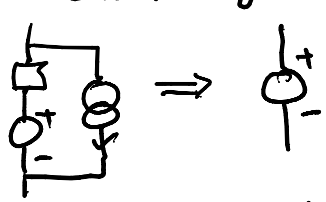
output

$$y = \begin{bmatrix} v_4 \\ v_5 \end{bmatrix}_b = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = Cx$$

Here $T(s) = C[sE - A]^{-1}B = Z(s)$

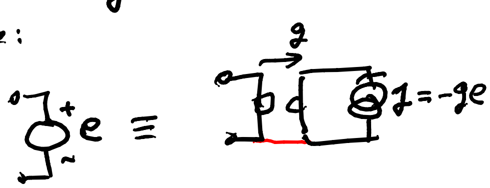
as $U(s)$ = currents, $Y(s)$ = outputs = voltages

But here had admittances in parallel with the input currents. If have voltage sources



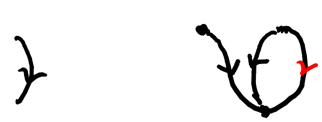
so using admittances is not so convenient

Still can always work with admittances in the linear case:



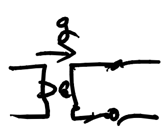
$i_1 = g v_2$
$i_2 = j = -g v_1$

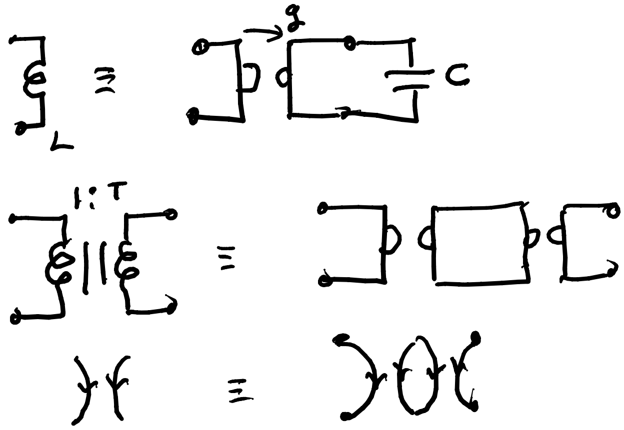
$v_1 = -j/g = e$



$\Rightarrow$ can replace voltage sources by current sources without changing $T(s)$ [does add 2 more branches to the graph]

also] a short $\equiv$



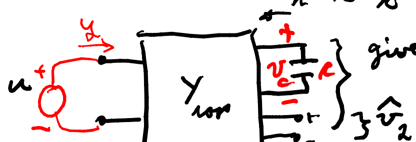


This means can assume a branch × branch admittance and derivatives only due to capacitors ⇒ gives semistate equations.

To reverse: someone gives us $E\dot{x} = Ax + Bu$

$y = Cx$

$i_c = c\dot{v}_c$



} gives left side } $y_{Load} = \begin{bmatrix} sc & 0 \\ 0 & 0 \end{bmatrix}$ } $\hat{v}_2$

} $\hat{v}_2$

$y_{in} = T(u)$; $\quad Y_{nor} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} v \\ \hline v_c \\ \hat{v}_2 \end{bmatrix}$ −

output → $\begin{bmatrix} i_{in} \\ i_c \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v \\ v_c \\ \hat{v}_2 \end{bmatrix} = \quad i_{in} = Y_{11} v + Y_{12} \begin{bmatrix} v_c \\ \hat{v}_2 \end{bmatrix}$

$$\begin{bmatrix} i_c \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v \\ \hline v_c \\ \hat{v}_2 \end{bmatrix}$$

derivative part of E → $\begin{bmatrix} c\dot{v}_c \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{21_v} & Y_{22_v} \\ Y_{21_{\hat{c}}} & Y_{22_{\hat{c}}} \end{bmatrix} \begin{array}{l} \begin{bmatrix} v \\ \hline v_c \\ \hat{v}_2 \end{bmatrix} \end{array} \begin{array}{l} \}u \\ \}x \end{array}$

implies can form Y_{nor} & make it by OTA's

⇒ any linear circuit can be made with OTA's and capacitors