EE 610
10/01/09

For $V^{T*} I$

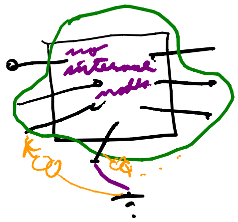
$$(V^{T*} I)^T = I^T V^* \neq I^{T*} V$$

Looking back

1) pull out all nodes & make ground external

2) ground one node

3) port admittance



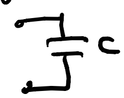
$\Rightarrow$ get the indefinite Y

$\Rightarrow$ get the nodal admittance

$\Rightarrow$ use only some nodes externally (eliminate "internal" nodes by setting external currents to zero).

Examples of $S(s)$

1)



$\Rightarrow y(s) = sC \Rightarrow$

$$z(s) = \frac{1}{y(s)} = \frac{1}{sC}$$

$$S = \frac{1-y}{1+y} = \frac{1-sC}{1+sC}$$; pole at $s = -1/C$

necessarily $C \geq 0$ if passive

$$1 - S^*(j\omega) S(j\omega) \geq 0 \text{ if passive}$$
$$= 1 - |S(j\omega)|^2 = 0$$

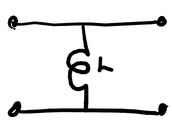
$$S(j\omega) = \frac{1 - j\omega C}{1 + j\omega C}$$

$$|S(j\omega)|^2 = \frac{1+\omega^2 C^2}{1+\omega^2 C^2} = 1$$

here C is real

$\therefore$ C is passive if $C \geq 0$

2)



$$Z(s) = sL \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}; \quad S = (Z + 1_2)^{-1}(Z - 1_2)$$

$$Z + 1_2 = \begin{bmatrix} 1+sL & sL \\ sL & 1+sL \end{bmatrix} \Rightarrow \det(Z + 1_2) = 1 + 2sL$$

$$S = \frac{1}{1+2sL} \begin{bmatrix} 1+sL & -sL \\ -sL & 1+sL \end{bmatrix} \begin{bmatrix} -1+sL & sL \\ sL & -1+sL \end{bmatrix} = \frac{1}{1+2sL} \begin{bmatrix} -1 & 2sL \\ 2sL & -1 \end{bmatrix}$$

$$(1+sL)(-1+sL) - (sL)^2 = -1 - sL + sL + (sL)^2 - (sL)^2 = -1$$

$$-sL(-1+sL) + (1+sL)sL = sL(1 - sL + 1 + sL) = 2sL$$

$$S(s) = \frac{1}{1+2sL} \begin{bmatrix} -1 & 2sL \\ 2sL & -1 \end{bmatrix}$$

assume $L \geq 0$

poles @ $s = -1/2L$

$$1_2 - S^{T*}(j\omega)S(j\omega) = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \frac{1}{1+4\omega^2L^2}\begin{bmatrix}-1 & -2j\omega L\\ -2j\omega L & -1\end{bmatrix}\begin{bmatrix}-1 & 2j\omega L\\ 2j\omega L & -1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \frac{1}{1+4\omega^2L}\begin{bmatrix}1+4\omega^2L^2 & 0\\ 0 & 1+4\omega^2L\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \equiv 0_2$$

here  is passive as S is bounded real

In these cases we had lossless circuits

$$\mathcal{E}(t)\Big|_{t\to\infty} = \int_{-\infty}^{\infty} v^T(t)\,i(\tau)\,d\tau = 0 \Leftrightarrow \text{definition of lossless}$$

$$\therefore \quad 1_m - S^{T*}(j\omega)S(j\omega) = 0_m \Rightarrow \text{lossless if passive}$$

$\mathcal{L}$ in $\mathcal{L}_2$, Lebesgue integral; Ex of non-$\mathcal{L}_2$ function is the unit-step function

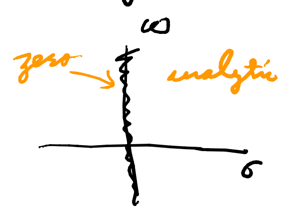
$$\int_{-\infty}^{\infty} 1^2(t)\,dt = \int_0^{\infty} 1^2\,dt = \infty$$

Ex of an $\mathcal{L}_2$ function is $s(t) = e^{-2t}1(t)$; $1(t) = \begin{cases}1 & t>0\\ 0 & t<0\end{cases}$

$$\int_{-\infty}^{\infty} s^2(t)\,dt = \int_{-\infty}^{\infty} e^{-4t}1(t)\,dt = \int_0^{\infty} e^{-4t}\,dt = \frac{1}{-4}e^{-4t}\Big|_{t=0}^{\infty}$$

$$= \frac{1}{4}e^{-0} = \frac{1}{4} < \infty$$

For lossless (passive) $\quad 1_m - S^{T*}(j\omega)S(j\omega) = 0_m$ for all ω if rational



$S^*(s) = S(s^*)$ if real components

$$\Rightarrow \quad 1_m - S^T(-j\omega)S(j\omega) = 0_m$$

analytically extend to $s = \sigma + j\omega$ with $\sigma > 0$

This means $\quad 1_m - S^T(-s)S(s) \equiv 0_m$ for all s

$\therefore$ lossless condion is $\quad 1_m = S^T(-s)S(s) \Rightarrow \underline{\underline{S^{-1}(s) = S^T(-s)}}$

Ex: $\quad S(s) = \dfrac{1-sC}{1+sC}$; $\quad S^{-1}(s) = \dfrac{1+sC}{1-sC}$

$S(-s) = \dfrac{1-(-s)C}{1+(-s)C} = \dfrac{1+sC}{1-sC} \quad \Leftarrow$ shows $S(-s) = \dfrac{1}{S(s)}$ for the capacitor

Ex:

$$S(s) = \frac{1}{1+2sL}\begin{bmatrix} -1 & 2sL \\ 2sL & -1 \end{bmatrix}$$

$$\det = \frac{-1}{1+2sL}\times\frac{-1}{1+2sL} - \frac{2sL}{1+2sL}\cdot\frac{2sL}{1+2sL} = \frac{1-(2sL)^2}{(1+2sL)^2} = \frac{1-2sL}{1+2sL}$$

$$S(-s) = \frac{1}{1-2sL}\begin{bmatrix} -1 & -2sL \\ -2sL & -1 \end{bmatrix} = S^T(-s)$$

$$S^{-1}(s) = \begin{bmatrix} \frac{-1}{1+2sL}\cdot\frac{1+2sL}{1-2sL} & \\ \frac{-2sL}{1+2sL} & \frac{1}{\frac{1-2sL}{1+2sL}} \end{bmatrix} = \begin{bmatrix} \frac{-1}{1-2sL} & \frac{-2sL}{1-2sL} \\ \frac{-2sL}{1-2sL} & \frac{-1}{1-2sL} \end{bmatrix}$$

here $S^T(-s)$ is $= S^{-1}(s)$ implies Σ_L is lossless

We have $S = (1_m + Y)^{-1}(1_m - Y)$; find conditions on Y to be lossless

$$1_m - \left((1_m+Y(s))^{-1}(1_m - Y(s))\right)^T\Big|_{s\to -s}\left((1_m+Y(s))^{-1}(1_m-Y(s))\right)$$

$$= 1_m - \underbrace{(1_m - Y^T(-s))(1_m+Y^T(-s))^{-1}}_{(1_m+Y^T(-s))^{-1}(1_m-Y^T(-s))}\underbrace{(1_m+Y(s))^{-1}(1_m-Y(s))}_{(1_m-Y(s))(1_m+Y(s))^{-1}}$$

$$= (1_m+Y^T(-s))^{-1}\left\{(1_m+Y^T(-s))(1_m+Y(s)) - (1_m - Y^T(-s))(1_m - Y(s))\right\}(1_m+Y(s))^{-1}$$

$$= (1_m+Y^T(-s))^{-1}\left\{\cancel{1_m} + Y^T(-s) + Y(s) + \cancel{Y^T(-s)\cdot Y(s)} - [\cancel{1_m} - Y(s) - Y^T(-s) + \cancel{Y^T(-s)Y(s)}]\right\}(1_m+Y(s))^{-1}$$

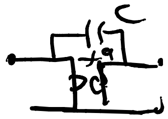
$$= 2(1_m+Y^T(-s))^{-1}\left\{Y(s)+Y^T(-s)\right\}(1_m+Y(s))^{-1} = 0_m \text{ if lossless}$$

$$\Rightarrow \quad Y(s) + Y^T(-s) = 0_m$$

if $m=1 \Rightarrow y(s)+y(-s) = 0 \Leftrightarrow Ev(y(s)) = 0$

Ex: cap.  $y = Cs \Rightarrow y(s)+y(-s) = C[s+(-s)] = 0$

and  $y = \frac{1}{Ls} \Rightarrow \frac{1}{Ls} + \frac{1}{L(-s)} = \frac{1}{L}\left(\frac{-s+s}{s(-s)}\right) = 0$



$$Y(s) = \begin{bmatrix} Cs & -Cs+g \\ -Cs-g & Cs \end{bmatrix}$$

$$Y^T(-s) = \begin{bmatrix} -Cs & Cs-g \\ Cs+g & -Cs \end{bmatrix}$$

add: $Y + Y_*^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Rightarrow$ this is a lossless circuit (very useful for synthesis)