EE 610
09/29/09

S: $AV = BI$

$\times C$ for which C^{-1} exists

$CAV = CBI$

if A^{-1} exists
$C = A^{-1}$ gives
$V = A^{-1}BI$
$Z = A^{-1}B \Rightarrow Y = B^{-1}A$

$$S = (B+A)^{-1}(B-A)$$
$$= (A[A^{-1}B + 1_m])^{-1}(A[A^{-1}B - 1_m])$$
$$= [Z + 1_m]^{-1}A^{-1}A[Z - 1_m] = [Z+1_m]^{-1}[Z-1_m]$$
$$= [Y+1_m]^{-1}[1_m - Y]$$

Given Y know S if $Y + 1_m$ is nonsingular

If know S to find Y:

$$[Y+1_m]S = 1_m - Y \Rightarrow YS + Y = 1_m - S$$
$$(S+1_m)Y = 1_m - S$$

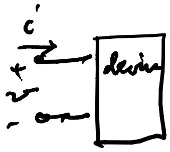
$$\Rightarrow \quad Y = (1_m+S)^{-1}(1_m - S)$$

look at $(1_m+S)(1_m-S) = 1_m + S - S - SS$

$(1_m - S)(1_m+S) = 1_m - S + S - SS$

$$(1_m+S)^{-1}[(\cancel{1_m+S})(1_m-S)](1_m+S)^{-1} = (1_m+S)^{-1}[(1_m-S)(\cancel{1_m+S})](1_m+S)^{-1}$$
$$(1_m-S)(1_m+S)^{-1} = (1_m+S)^{-1}(1_m-S)$$

$\therefore$ $(1_m+S)^{-1}$ commutes with $(1_m - S)$

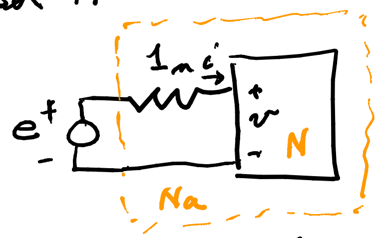
Passivity: 

, $p_{in}(t) = v(t)^T i(t)$ = instantaneous

$$\mathcal{E}(t) = \int_{-\infty}^{t} p_{in}(\tau)\,d\tau = \int_{-\infty}^{t} v(\tau)^T i(\tau)\,d\tau = \text{energy into the device}$$

≥ 0 if passive at every t (including ∞)

For the terminated N



P_{in_N} , $P_{in_{Na}}$, $2v^i = v + i$, $2v^r = v - i$

look at $e(t) \in \mathcal{L}_{2_m} \Rightarrow \int_{-\infty}^{\infty} \underbrace{e^T(\tau)e(\tau)}_{\sum_{i=1}^{n} e_i^2} d\tau < \infty$

$e = v + i = 2v^i$

$4v^{i^T}v^i = e^T e = (v+i)^T(v+i) = v^Tv + i^Ti + i^Tv + v^Ti = v^Tv + i^Ti + 2v^Ti$

$4v^{r^T}v^r = (v-i)^T(v-i) = v^Tv + i^Ti - i^Tv - v^Ti = v^Tv + i^Ti - 2v^Ti$

here $v^Ti = i^Tv$

$P_{in_{NA}}$: $e^Ti = (v+i)^Ti = v^Ti + i^Ti$

$$\left.\begin{aligned} 4v^{i^T}v^i &= v^Tv + i^Ti + 2v^Ti \\ 4v^{r^T}v^r &= v^Tv + i^Ti - 2v^Ti \end{aligned}\right\} \quad 4\left(v^{i^T}v^i - v^{r^T}v^r\right) = 4v^Ti$$

if $e \in \mathcal{L}_2 \Rightarrow v^i \in \mathcal{L}_2$ if N is passive then $\int_{-\infty}^{\infty} v^T(\tau)i(\tau)d\tau \geq 0$

$$\underbrace{\int_{-\infty}^{\infty} v^{i^T}(\tau)v^i(\tau)d\tau}_{\geq 0} - \underbrace{\int_{-\infty}^{\infty} v^{r^T}(\tau)v^r(\tau)d\tau}_{\text{implies this exists}} = \int_{-\infty}^{\infty} v^T(\tau)i(\tau)d\tau \geq 0$$

$\int_{-\infty}^{\infty} v^{r^T}(\tau)v^r(\tau)\,d\tau \geq 0$ & exists

if $e \in \mathcal{L}_{2_m} \Rightarrow v^i$ & v^r are in $\mathcal{L}_{2_m}$ this mean $S(s)$ exists and Na maps $\mathcal{L}_{2_m}$ signals into $\mathcal{L}_{2_m}$ signals

To see $S(s)$ exists use Fourier transforms and Parsevals relationship between $v^i(t)$ & $V^i(j\omega)$,

$$\int_{-\infty}^{\infty} g^{T*}(t)\,h(t)\,dt = \int_{-\infty}^{\infty} \mathcal{F}_f[g]^{T*} \cdot \mathcal{F}_f[h]\,df$$

$$|v^i|^2_{\mathcal{L}_2} = \int_{-\infty}^{\infty} v^{iT*}(\tau)\, v^i(\tau)\, d\tau = \int_{-\infty}^{\infty} V^{iT*}(j\omega)\, V^i(j\omega)\, d\frac{\overset{\omega}{2\pi f}}{2\pi}$$

$$\int v^{rT*} v^r d\tau = \int V^{rT*} V^r df$$

$$\int (v^{iT*} v^i - v^{rT*} v^r)\, d\tau = \int_{-\infty}^{\infty} V^{iT*} V^i - V^{rT*} V^r df$$

but $V^r(j\omega) = S(j\omega)\, V^i(j\omega)$

$$\Rightarrow = \int \left(V^{i*T} V^i - V^{i*T} S^{T*}(j\omega)\, S\, V^i\right) \frac{d\omega}{2\pi} \geq 0 \text{ if passive}$$

∴ $S(j\omega)$ exists if N is passive (if can apply $\mathcal{L}_2$ signals)

But we work with $s = \sigma + j\omega$
so what happens in $S(s)$:
∴ no poles on $s = j\omega$ axis as
$|V^i|^2 - |SV^i|^2 \geq 0$ on $s = j\omega$
& in fact $S(s)$ is analytic (almost everywhere) on $s = j\omega$ (if passive)

also if passive no poles in Re $\sigma > 0$ as no unstable natural frequencies for Na

extend by analytic continuation to $\sigma \geq 0$; i.e., $S(s)$ is "known" for $s = \sigma + j\omega$, $\sigma \geq 0$

also if the circuit has only real components $S(\sigma)$ is real for $\sigma \geq 0 \Rightarrow S^*(s^*) = S(s)$

also

$$|V^i(j\omega)|^2 - |S(j\omega) V^i(j\omega)|^2 = |V^i|^2 - \|S(j\omega)\|^2 |V^i|^2 \geq 0$$

$$1 - \|S(j\omega)\|^2 \geq 0 \qquad \text{real passive constraint}$$

$$1 \geq \|S(j\omega)\|^2$$

Conditions for a circuit (n-port/terminals) to be passive:
a) $S(s)$ exists for $\sigma \geq 0$
b) $S(\sigma)$ is real
c) $1 \geq \|S(j\omega)\|^2$
d) $S(s)$ is analytic in $\sigma \geq 0$

These are the bounded real conditions:

also if $S(s)$ is rational we can make a passive N with R, L, C, gyrators, transformers (actually just R, C, gyrators)

But are used to impedance and admittance in a

We have
$$|V^i|^2 - |SV^i|^2 = V^{iT*}V^{i*T} - V^{iT*}S^{*T}(j\omega)S(j\omega)V^i$$

$$= V^{iT*}\left(1_n - S^{T*}(j\omega)S(j\omega)\right)V^i \geq 0$$ if passive for "almost all" ω & all $V^i(j\omega)$

$2V^i = V + I$

$2V^r = V - I$

$$= V^{iT*}V^i - V^{rT*}V^r$$

$$= \tfrac{1}{4}(V+I)^{T*}(V+I) - 4(V-I)^{T*}(V-I)$$

$$= \tfrac{1}{4}\{\cancel{V^{T*}V} + \cancel{I^{T*}I} + \cancel{2}V^{T*}I + I^{T*}V$$
$$- \cancel{V^{T*}V} - \cancel{I^{T*}I} + \cancel{2}V^{T*}I + I^{T*}V\}$$

note correction

$$= \frac{V^{T*}I + I^{T*}V}{2} = \operatorname{Re} V^{T*}I$$

$$= \frac{V^{T*}Y(j\omega)V + V^{T*}Y^{*T}(j\omega)V}{2} = \frac{V^{T*}\left(Y(j\omega) + Y^{T*}(j\omega)\right)V}{2}$$

for a 1-port
$$= V^*\frac{(y + y^*)}{2}V = \frac{|V|^2}{2}\operatorname{Re} y(j\omega) \geq 0$$ if passive

$\Rightarrow$ $Y(s)$ is positive-real

& $Z(s)$ is also by duality