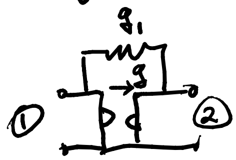
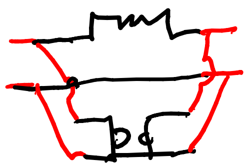
Necessary & sufficient

EE610
09/22/09



$$Y = \begin{bmatrix} g_1 & -g_1 \\ -g_1 & g_1 \end{bmatrix} + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} = \begin{bmatrix} g_1 & g-g_1 \\ -g-g_1 & g_1 \end{bmatrix}$$

$g_1 \geq 0$, g is real



Given $Y(s) = \begin{bmatrix} y_{11}(s) & y_{12}(s) \\ y_{21}(s) & y_{22}(s) \end{bmatrix}$ which goes with the above circuit

necessarily $Y(s)$ is independent of s & all entries are real (not yet sufficient)

Rewrite $Y = Y_{sy} + Y_{ss}$; $Y_{sy} = \frac{Y+Y^T}{2}$, $Y_{ss} = \frac{Y-Y^T}{2}$

here necessarily Y_{sy} has rank 1 with $y_{11_{sy}} = y_{22_{sy}}$

$$Y_{sy} = \begin{bmatrix} y_{11_{sy}} & y_{12_{sy}} \\ y_{12_{sy}} & y_{11_{sy}} \end{bmatrix} \text{ as rank } Y_{sy} = 1, \det Y_{sy} = 0 = y_{11_{sy}}^2 - y_{12_{sy}}^2 = 0$$

$$\Longrightarrow y_{12_{sy}}^2 = y_{11_{sy}}^2 \Rightarrow y_{12_{sy}} = \pm\sqrt{y_{11_{sy}}^2} = \pm y_{11_{sy}}$$

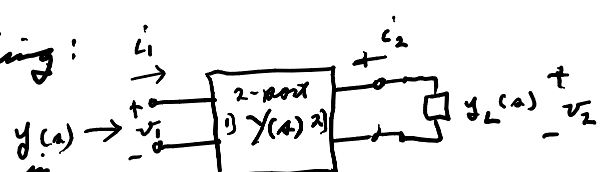
we need $y_{11_{sy}} = g \geq 0$ & then we need $y_{12_{sy}} = -y_{11_{sy}}$

∴ necessarily, Y is real, independent of s,

$$y_{11_{sy}} = y_{22_{sy}} = -y_{12_{sy}} \geq 0$$

these are also sufficient since given a $Y(s)$ satisfying these conditions we can make the circuit.

2-port loading:



$i_2 = -y_L v_2$

$i_1 = y_{in} v_1$

$$\begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

2nd eq: $-y_L v_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow (-y_L - y_{22}) v_2 = y_{21} v_1$

$$v_2 = -(y_L + y_{22})^{-1} y_{21} v_1$$

1st eq. $i_1 = y_{11} v_1 + y_{12} v_2 = y_{11} v_1 - y_{12}(y_L + y_{22})^{-1} y_{21} v_1$

$$y_{in} = y_{11} - y_{12}(y_L + y_{22})^{-1} y_{21}$$

for a 2-port, $y_L + y_{22}$ is 1x1 so can divide

$$y_{in} = \frac{y_{11}(y_L + y_{22}) - y_{12} y_{21}}{y_L + y_{22}} = \frac{y_L y_{11} + \Delta_y}{y_L + y_{22}}, \quad \Delta_y = \text{det of } Y$$

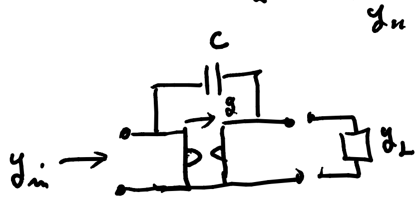
Let's also find y_L as a function of y_{in}

$$y_{in} \cdot y_L + y_{in} \cdot y_{22} = y_L y_{11} + \Delta y$$

$$(y_{in} - y_{11}) y_L = -y_{in} y_{22} + \Delta y$$

$$y_L = \frac{y_{in} y_{22} - \Delta y}{y_{11} - y_{in}}$$

Ex:



$$Y = \begin{bmatrix} sC & -sC + g \\ -sC - g & sC \end{bmatrix}$$

$$\det Y = (sC)^2 - (-sC + g)(-sC - g)$$
$$= \cancel{(sC)^2} - (\cancel{s^2C^2} + \cancel{sCg} - \cancel{sCg} - g^2) = g^2$$

$$y_L = \frac{y_{in} sC - g^2}{sC - y_{in}}$$

continue with a given $y_{in}(s) = \frac{s}{s^2 + 2}$ is degree 2

the load to give this is

$$y_L(s) = \frac{\frac{s}{s^2+2} \cdot sC - g^2}{sC - \frac{s}{s^2+2}} = \frac{s^2 C - g^2 s^2 - 2g^2}{s^3 C + 2sC - s}$$

$$= \frac{(C - g^2)s^2 - 2g^2}{s[s^2 C + 2C - 1]}$$ looks to be degree 3

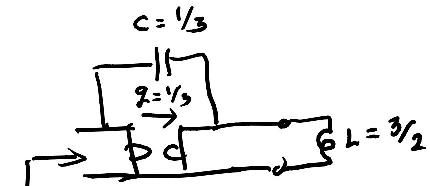
force a pole at $s = 1$; $s(s^2 C + 2C - 1)\Big|_{s=1} = 0 = (s-1)(s^2 + \cdots$

$$1(C + 2C - 1) = 0 \Rightarrow C = 1/3$$

force a zero at $s = 1$ $\quad (1/3 - g^2)s^2 - 2g^2\Big|_{s=1} = 0 \Rightarrow \frac{1}{3} - 3g^2 = 0$
$$\Rightarrow g = \pm 1/3$$

$$y_L(s) = \frac{(1/3 - 1/9)s^2 - 2/9}{s(s^2\frac{1}{3} + \frac{2}{3} - 1)} = \frac{\frac{2}{9}s^2 - 2/9}{s(\frac{1}{3}s^2 - 1/3)} = \frac{\frac{2}{9}(s^2-1)}{\frac{1}{3}s(s^2-1)}$$

$$= \frac{2}{3s}$$

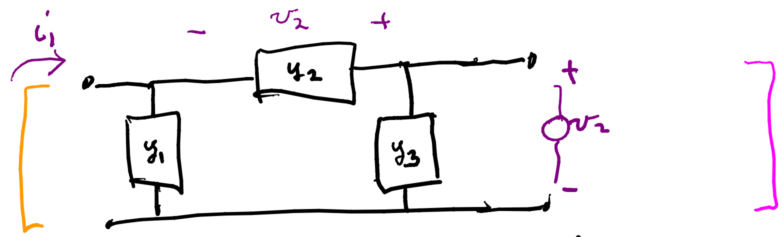


choose C & g to get y_L simpler than y_{in}

$$y_{in} = \frac{s}{s^2+2}$$

2-port π equivalent: $Y(s)$

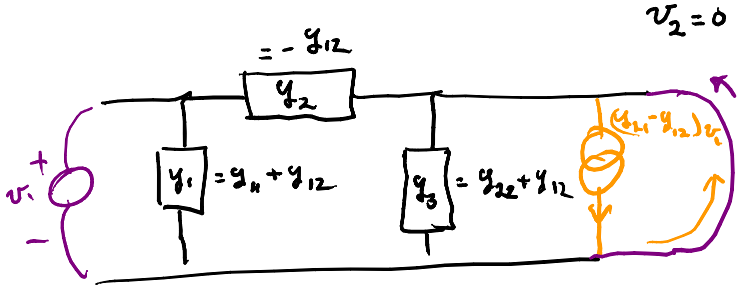
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = y_1 + y_2, \quad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = y_2 + y_3$$

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -y_2 \implies y_{21} = -y_2$$ but needs y_{12} possibly $\neq y_{21}$

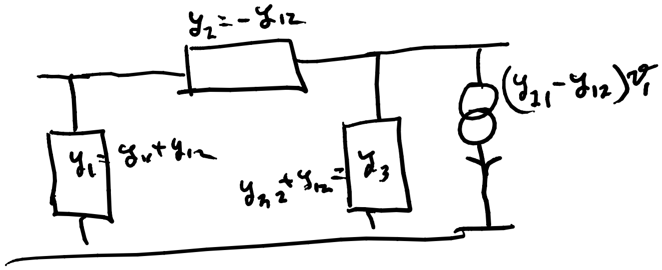
$$= \left.\frac{i_2}{v_1}\right|_{v_2=0}$$



$i_2 = -(-y_{12})v_1 +$ correction

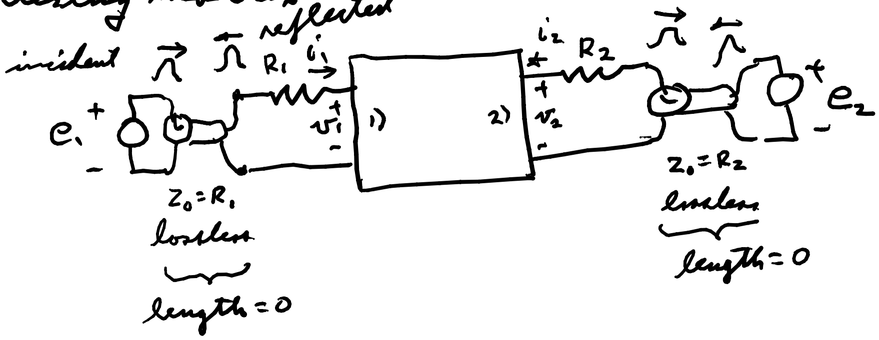
$= y_{21} v_1$

correction $= (y_{21} - y_{12})v_1$



= π equivalent of a 2-port

scattering matrix



v^i = incident voltage (2-vector)

v^r = reflected voltage (2-vector)

S = scattering matrix $\Rightarrow$ $v^r = S v^i$

$e_1 = v_1 + R_1 i_1$, $e_2 = v_2 + R_2 i_2$, $e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$; $R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}$

$\frac{1}{2} e = v^i \Rightarrow \left.\begin{array}{l} 2v^i = v + Ri \\ 2v^r = v - Ri \end{array}\right\} \begin{array}{l} 2v = 2v^i + 2v^r \Rightarrow v = v^i + v^r \\ 2Ri = 2v^i - 2v^r \Rightarrow Ri = v^i - v^r \end{array}$

$2v^r$