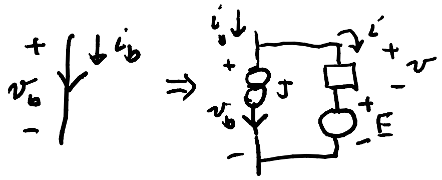
EE610
09/17/09

finite (have a bounded graph)



$i_b = J + i$
$v_b = E + v$

$v_b = \begin{bmatrix} v_t \\ v_\ell \end{bmatrix}$, $i_b = \begin{bmatrix} i_t \\ i_\ell \end{bmatrix}$

KCL: $0_t = \mathcal{C} v_b$, KVL: $0_\ell = \mathcal{J} i_b$
$i_b = \mathcal{J}^T i_\ell$ $\quad\quad$ $v_b = \mathcal{C}^T v_t$

Linear time-invariant,
$$A v = B i$$
(assume A & B are $b \times b$)

Use these: $i = i_b - J$, $v = v_b - E$

$$A[v_b - E] = B[i_b - J] \Rightarrow A v_b - B i_b = \underbrace{AE - BJ}_{\text{source term}}$$

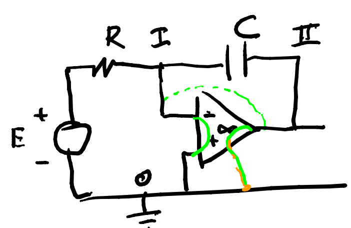
$$\Rightarrow A \mathcal{C}^T v_t - B \mathcal{J}^T i_\ell = AE - BJ$$

$b = t + \ell$

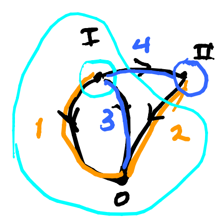
$$\underbrace{\begin{bmatrix} A\mathcal{C}^T & -B\mathcal{J}^T \end{bmatrix}}_{b \times b} \underbrace{\begin{bmatrix} v_t \\ i_\ell \end{bmatrix}}_{b \times 1} = \underbrace{AE - BJ}_{b \times 1}$$

} b equations
& in general b unknowns

Ex:



$\Rightarrow$



$b = 4$
$t = 2$, $\ell = 2$

tree
links
||
co-tree

cut set eqs: $t\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right. = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}_b$

$1_t = 1_2$

$-(\cdot)^T$

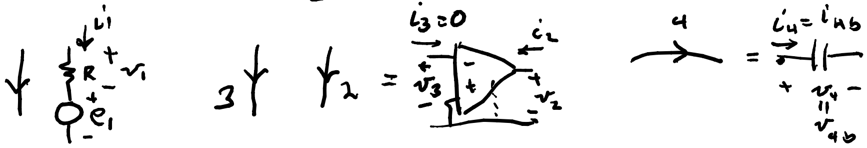
tie set eqs: $\ell\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right. = \begin{bmatrix} -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}_b$

$1_\ell = 1_2$

$\mathcal{C}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix}$, $\mathcal{J}^T = \begin{bmatrix} -1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

$P_{\text{ai}} = v_b^T i_b = v_t^T \mathcal{C}_1 \mathcal{J}_2^T i_{\ell_2} = 0$
$\Rightarrow \mathcal{C}_1 \mathcal{J}_2^T = 0_{t \times \ell}$

$$Av = Bi \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & AC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$



$i_4 = AC \cdot v_4$

as $\det A = 0$ & $\det B = 0$ no Z or Y matrix for this "graph"

$[AC^T \quad -BJ^T]$

$$AC^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & AC \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ AC & -AC \end{bmatrix} ; \quad BJ^T = \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -R & -R \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad J = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \qquad AE = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & AC \end{bmatrix} \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & R & R \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ AC & -AC & 0 & -1 \end{array}\right] \begin{bmatrix} v_{t1} \\ v_{t2} \\ i_{x3} \\ i_{x4} \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

when solve will obtain all voltages & currents of the graph

$\Rightarrow -i_{\ell_3} = 0$ (3rd eq.) $\Rightarrow$ can eliminate row & column 3

$$\begin{bmatrix} 1 & 0 & R \\ 1 & 0 & 0 \\ AC & -AC & -1 \end{bmatrix} \begin{bmatrix} v_{t1} \\ v_{t2} \\ i_{x4} \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \\ 0 \end{bmatrix}$$

This has $v_{t_1} = 0$; $\quad 0 = v_{t_1} = -R\, i_{x4} + e_1 \Rightarrow i_{x4} = \frac{1}{R} e_1$

$i_{\ell_4} = AC\, v_{t_1} - AC\, v_{t2} \Rightarrow \quad v_{t_2} = -\frac{1}{AC} i_{x_4}$