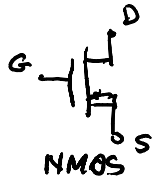
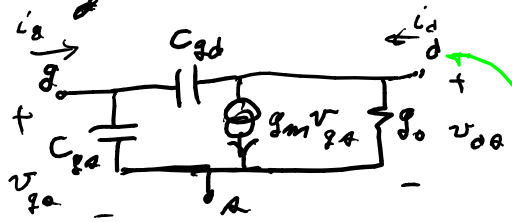
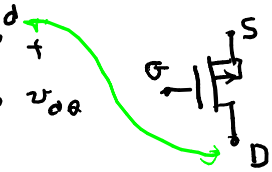
Homework due today now due Th   EE610 09/15/09



$\Rightarrow$

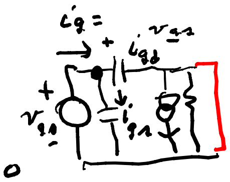




$$\begin{bmatrix} i_g \\ i_d \end{bmatrix} = Y \begin{bmatrix} v_{gs} \\ v_{ds} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_{gs} \\ v_{ds} \end{bmatrix}$$

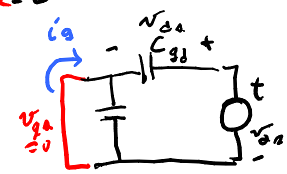
$$i_g = y_{11} v_{gs} + y_{12} v_{ds} \Rightarrow y_{11} = i_g / v_{gs} \Big|_{v_{ds}=0}$$

$$= (sC_{gs} + sC_{gd})$$



$$Y = \begin{bmatrix} sC_{gs} + sC_{gd} & -sC_{gd} \\ g_m - sC_{gd} & g_o + sC_{gd} \end{bmatrix}$$

$$y_{12} = i_g / v_{ds} \Big|_{v_{gs}=0}$$

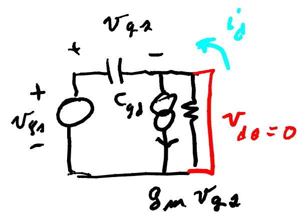


$$= -sC_{gd}$$

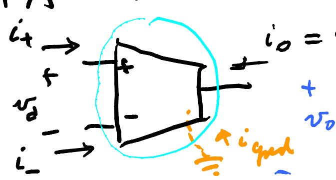
$$i_d = y_{21} v_{gs} + y_{22} v_{ds}$$

$$y_{21} = \frac{i_d}{v_{gs}} \Big|_{v_{ds}=0}$$

$$= g_m - sC_{gd}$$



OTA = operational transconductance amplifier



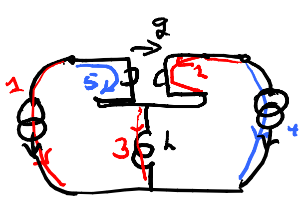
$$i_o + i_+ + i_- = 0 \quad \text{by KCL}$$

$+ i_{gnd}$

$$i_+ = i_- = 0$$

$$i_{in} = i_+ \text{ or } i_-$$

$$\begin{bmatrix} i_{in} \\ i_o \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \begin{bmatrix} v_d \\ v_o \end{bmatrix}$$



$$Y_{b \times b} = Y_{5 \times 5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g \\ 0 & 0 & \frac{1}{sL} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -g & 0 & 0 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} J_1 \\ 0 \\ 0 \\ J_4 \\ 0 \end{bmatrix}, \quad E = 0_5$$

$$e = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

form

$$v_t = [e Y_{b \times b} e^T]^{-1} \{ e (Y_{b \times b} E - J) \}$$

$$-eJ = -\begin{bmatrix}1&0&0&0&1\\0&1&0&1&0\\0&0&1&1&-1\end{bmatrix}\begin{bmatrix}J_1\\0\\0\\J_4\\0\end{bmatrix} = -\begin{bmatrix}J_1\\J_4\\J_4\end{bmatrix}$$

$$eY_{b\times b}e^T = \begin{bmatrix}1&0&0&0&1\\0&1&0&1&0\\0&0&1&1&-1\end{bmatrix}\begin{bmatrix}0&0&0&0&0\\0&0&0&0&g\\0&0&\frac{1}{sL}&0&0\\0&0&0&0&0\\0&-g&0&0&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&1&1\\1&0&-1\end{bmatrix}$$

$$= \begin{bmatrix}0&-g&0&0&0\\0&0&0&0&g\\0&g&\frac{1}{sL}&0&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&1&1\\1&0&-1\end{bmatrix} = \begin{bmatrix}0&-g&0\\g&0&-g\\0&g&\frac{1}{sL}\end{bmatrix}$$

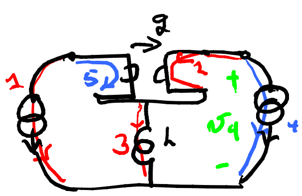
$$v_t = \begin{bmatrix}0&-g&0\\g&0&-g\\0&g&\frac{1}{sL}\end{bmatrix}^{-1}\begin{bmatrix}-J_1\\-J_4\\-J_4\end{bmatrix}; \quad (eY_{b\times b}e^T)^{-1}; \quad \det eY_{bb}e^T = -\frac{1}{sL}(-g^2) = g^2/sL$$

Cofactors: $\Delta_{11} = g^2$, $\Delta_{21} = \left(-\frac{g}{sL}\right)(-1)^{2+1} = g/sL$, $\Delta_{31} = g^2\times(-1)^{3+1} = g^2$

$\Delta_{12} = \frac{g}{sL}(-1)^{1+2} = \frac{-g}{sL}$, $\Delta_{22} = 0$, $\Delta_{32} = 0$

$\Delta_{13} = g^2$, $\Delta_{23} = 0$, $\Delta_{33} = g^2$

$$(eY_{b\times b}e^T)^{-1} = \frac{sL}{g^2}\begin{bmatrix}g^2&g/sL&g^2\\-g/sL&0&0\\g^2&0&g^2\end{bmatrix} = \begin{bmatrix}sL&1/g&sL\\-1/g&0&0\\sL&0&sL\end{bmatrix}$$

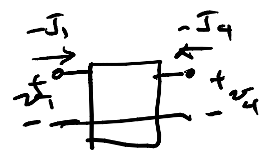


$$\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} = \begin{bmatrix}sL&1/g&sL\\-1/g&0&0\\sL&0&sL\end{bmatrix}\begin{bmatrix}-J_1\\-J_4\\-J_4\end{bmatrix}$$

$$v_4 = v_2 + v_3 = \begin{bmatrix}-\frac{1}{g}+sL & 0 & sL\end{bmatrix}\begin{bmatrix}-J_1\\-J_4\\-J_4\end{bmatrix} = \begin{bmatrix}-\frac{1}{g}+sL & sL\end{bmatrix}\begin{bmatrix}-J_1\\-J_4\end{bmatrix}$$

$$v_1 = \begin{bmatrix}sL & \frac{1}{g}+sL\end{bmatrix}\begin{bmatrix}-J_1\\-J_4\end{bmatrix}$$

2-port description $v_{2port} = \begin{bmatrix}v_1\\v_4\end{bmatrix}$, $i_{2port} = \begin{bmatrix}-J_1\\-J_4\end{bmatrix}$



$$\begin{bmatrix} v_{1_{2port}} \\ v_{2_{2port}} \end{bmatrix} = \begin{bmatrix} sL & \frac{1}{g}+sL \\ -\frac{1}{g}+sL & sL \end{bmatrix} \begin{bmatrix} i_{1_{2port}} \\ i_{2_{2port}} \end{bmatrix} \; ; \quad Z_{2port} = \begin{bmatrix} sL & \frac{1}{g}+sL \\ -\frac{1}{g}+sL & sL \end{bmatrix}$$

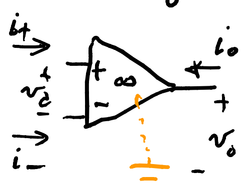
$$\underbrace{Y}_{\text{for the 2-port}} = Z^{-1} = \frac{1}{\det} \begin{bmatrix} \Delta_{11} & \Delta_{21} \\ \Delta_{12} & \Delta_{22} \end{bmatrix} = \frac{1}{1/g^2} \begin{bmatrix} sL & -\frac{1}{g}-sL \\ \frac{1}{g}-sL & sL \end{bmatrix}$$

Given a square matrix A we can form its symmetric, A_{sy}, and skew symmetric, A_{sk}, parts to give $A = A_{sy} + A_{sk}$

$$A_{sy} = \frac{A + A^T}{2} = A_{sy}^T \; , \quad A_{sk} = \frac{A - A^T}{2} = -A_{sk}^T$$

$$Y_{sy} = \frac{1}{1/g^2} \begin{bmatrix} sL & -sL \\ -sL & sL \end{bmatrix} \; , \quad Y_{sk} = \frac{1}{1/g^2} \begin{bmatrix} 0 & -1/g \\ \frac{1}{g} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$$

What to do if no branch by branch $Y_{b \times b}$?



ideal op amp

$$v_d \equiv 0 \; ; \; i_{in} = i_+ \text{ or } i_- \equiv 0$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_d \\ v_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_{in} \\ i_o \end{bmatrix}$$

Here $\underbrace{Av = Bi}_{\text{general linear description of circuit components}}$; if Y exists $\Rightarrow i = Yv$
then B^{-1} exists & $Y = B^{-1}A$

if Z exists $\Rightarrow v = Zi = A^{-1}Bi$

$$Z = A^{-1}B$$