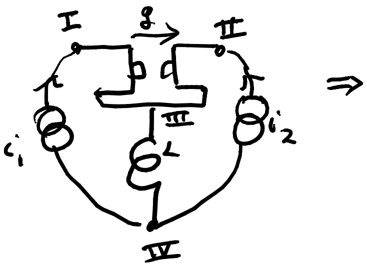
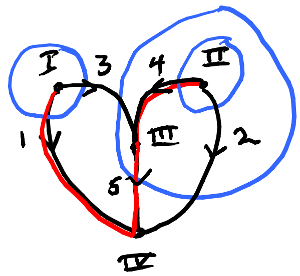
EE610
09/10/09



$\Rightarrow$



$b = 5$
$n = 4$
$t = 3$

—— = tree

KCL $\Rightarrow$ $\mathcal{C}$ = cut-set matrix
KVL $\Rightarrow$ $\mathcal{T}$ = tie set matrix

KCL (node 1) $0 = i_1 + i_3$
" 4) $0 = +i_2 + i_4$
" 5) $0 = +i_2 - i_3 + i_5$

$$\Rightarrow \begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 1 & 0\\ 0 & 1 & -1 & 0 & 1\end{bmatrix}\begin{bmatrix}i_1\\i_2\\i_3\\i_4\\i_5\end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix}1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 1 & 0\\ 0 & 1 & -1 & 0 & 1\end{bmatrix}$$

KVL 2) $0 = +v_2 - v_4 - v_5$
3) $0 = -v_1 + v_3 + v_5$

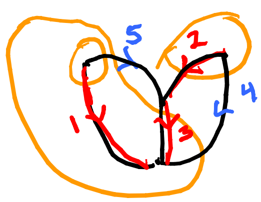
$$\Rightarrow \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & -1 & -1\\ -1 & 0 & 1 & 0 & 1\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{bmatrix}$$

$$\mathcal{T} = \begin{bmatrix}0 & 1 & 0 & -1 & -1\\ -1 & 0 & 1 & 0 & 1\end{bmatrix}$$

$\mathcal{C}$ is $t \times b$, $\mathcal{T}$ is $l \times b$

$b = t + l$

By renumbering we can make the 1st t columns of $\mathcal{C}$ to be the identity matrix, 1_t
then for this tree $\mathcal{T}$ has the last l columns to be 1_l



$$\mathcal{C} = \left[\begin{array}{ccc|cc}1 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 1 & 0\\ 0 & 0 & 1 & 1 & -1\end{array}\right] = \left[\begin{array}{c|cc} 1_3 & \begin{matrix}0 & 1\\ 1 & 0\\ 1 & -1\end{matrix}\end{array}\right] = [1_t \mid K_c]$$

$$\mathcal{T} = \left[\begin{array}{ccc|cc}0 & -1 & -1 & 1 & 0\\ -1 & 0 & 1 & 0 & 1\end{array}\right] = \left[\begin{array}{ccc|c}0 & -1 & -1 & \\ -1 & 0 & 1 & 1_2\end{array}\right] = [K_T \mid 1_l]$$

here $K_T = -K_c^T$ if true for all finite circuits then KVL $\Leftrightarrow$ KCL ; it is true

KCL $0_t = [1_t \mid K_c]\begin{bmatrix}i_t\\i_l\end{bmatrix} = \mathcal{C} i_b$

$i_b = \begin{bmatrix}i_t\\i_l\end{bmatrix}$

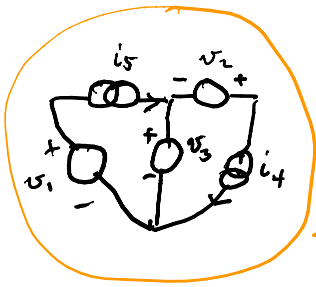
KVL: $0_l = [K_T \mid 1_l]\begin{bmatrix}v_t\\v_l\end{bmatrix} = \mathcal{T} v_b$

$-i_t = K_c i_l$

$-v_l = K_T v_t$; $v_b = \begin{bmatrix}v_t\\v_l\end{bmatrix} = \begin{bmatrix}1_t\\-K_T\end{bmatrix} v_t$

$i_b = \begin{bmatrix}i_t\\i_l\end{bmatrix} = \begin{bmatrix}-K_c\\1_l\end{bmatrix} i_l$

for any circuit with graph this holds; choose one with arbitrary v_t
here i_l can be arbitrary



here can choose v_t anything & i_ℓ anything

← power into here $\equiv 0$

super ^{T} = transpose

$$P_{in} = 0 = v_b^T i_b = [v_1\ v_2 \cdots v_b]\begin{bmatrix} i_1 \\ \vdots \\ i_b \end{bmatrix} = 0$$

$$= [v_t^T\ v_\ell^T]\begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = v_t^T i_t + v_\ell^T i_\ell = 0$$

$$= v_t^T [1_t \,\vdots\, -K_T^T]\begin{bmatrix} -K_c \\ \hline 1_\ell \end{bmatrix} i_\ell = v_t^T \underbrace{[-K_c - K_T^T]}_{t\times\ell} i_\ell = 0$$

as v_t & i_ℓ are arbitrary $\Rightarrow -K_c - K_T^T = 0_{t\times\ell} \Rightarrow K_T^T = -K_c$

$\equiv K_T = -K_c^T$

here $P_{in} = v_t^T e^{\cancel{T}} \mathcal{J}^T i_\ell = 0 \Rightarrow e^{\cancel{T}}\mathcal{J}^T = 0_{t\times\ell}$

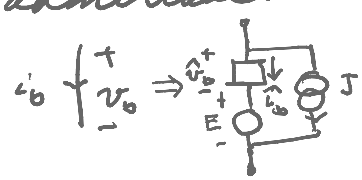
where $v_b = e^T v_t$, $i_b = \mathcal{J}^T i_\ell$; $\underline{o}_t = e\, i_b$, $\underline{o}_\ell = \mathcal{J} v_b$

4a,b,c,d)

Use these to set up circuit equations where we have a finite graph

Use the branch by branch admittance

1) $\hat{i}_b = Y_{b\times b}\hat{v}_b$



2) $i_b = \hat{i}_b + J$

3) $v_b = \hat{v}_b + E$

$$i_b \overset{1\to 2)}{=} Y_{b\times b}\hat{v}_b + J \overset{3)}{=} Y_{b\times b}(v_b - E) + J$$

$$= Y_{b\times b} v_b + \underbrace{(J - Y_{b\times b}E)}$$ Norton's equivalent branch current sources

× 4c) & use 4a)

$$e\, i_b = \underline{o}_t = e\, Y_{b\times b} e^T v_t + e\,(J - Y_{b\times b}E)$$

here there are t equations in t unknowns v_t

$$e(Y_{b\times b}E - J) = \left[eY_{b\times b}e^T\right]_{t\times t} v_t$$

$$\Rightarrow \quad v_t = \left[eY_{b\times b}e^T\right]^{-1}\left\{e(Y_{b\times b}E - J)\right\}$$

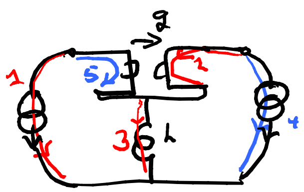
$$\& \quad v_b = e^T v_t$$

$$\hat{v}_b = v_b - E \quad \Rightarrow \quad \hat{i}_b = Y_{b\times b}\,\hat{v}_b$$

$$i_b = \hat{i}_b + J$$

$\therefore$ know all voltages & currents if can invert $eY_{b\times b}e^T$

Ex



$$Y_{b\times b} = Y_{5\times 5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g \\ 0 & 0 & \frac{1}{\Delta h} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -g & 0 & 0 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} J_1 \\ 0 \\ 0 \\ J_4 \\ 0 \end{bmatrix}, \quad E = \underline{0}_5$$

$$e = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$