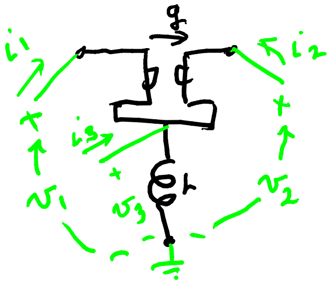
obtain a paper by next period

EE 610
09/08/09



$\Rightarrow$ nodal admittance

$i_{nod} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$, $v_{nod} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$, $i_{node} = Y_{nodal} v_{nodal}$

$i_1 = g(v_2 - v_3)$

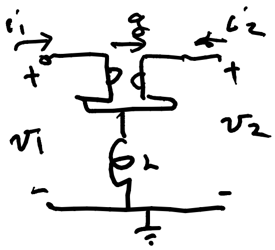
$i_2 = -g(v_1 - v_3)$

$i_3 = -(i_1 + i_2) + i_L = -g(v_2 - v_3) - (-g)(v_1 - v_3) + \frac{1}{sL} v_3$

KCL

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_{node} = \begin{bmatrix} 0 & g & -g \\ -g & 0 & g \\ g & -g & \frac{1}{sL} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_{node} ; \quad Y_{node}(s) = \begin{bmatrix} 0 & g & -g \\ -g & 0 & g \\ g & -g & 1/sL \end{bmatrix}$$

For terminal (2-port) Y



$\Rightarrow$ "eliminate" node 3

$i_{3\,node} = 0$

$$\begin{bmatrix} i_1 \\ i_2 \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & y_{12} \\ \hline y_{21} & y_{22} \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$0 = y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + y_{22} v_3 = [g \;\; -g] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \left[\frac{1}{sL}\right][v_3]$$

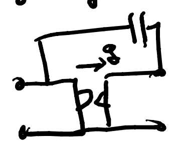
$$v_3 = -y_{22}^{-1} y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -sL[g \;\; -g] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = [Y_{11} \;|\; y_{12}] \begin{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \\ [-sLg \;\; +sLg] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \end{bmatrix} = [Y_{11} \;|\; y_{12}] \begin{bmatrix} \begin{matrix} v_1 \\ v_2 \end{matrix} \\ \hline -y_{22}^{-1} y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \end{bmatrix}$$

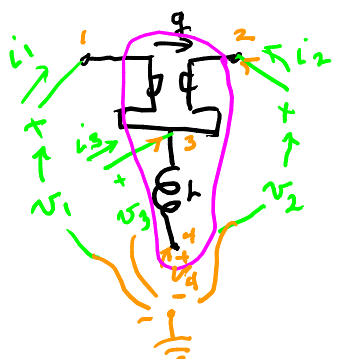
$$= \left\{ \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} -g \\ g \end{bmatrix} [-sLg \;\; sLg] \right\} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [Y_{11} - y_{12} y_{22}^{-1} y_{21}] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{\text{terminal}} = Y_{11} - y_{12}\, y_{22}^{-1}\, y_{21}$$

$$= \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} sLg^2 & -sLg^2 \\ -sLg^2 & sLg^2 \end{bmatrix} = \begin{bmatrix} sLg^2 & g-sLg^2 \\ -g-sLg^2 & sLg^2 \end{bmatrix}$$



Indefinite $Y = Y_{ind}$



ground is outside the circuit
now get a 4×4 Y_{ind}

$\underset{ind}{\underline{i}} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$, $\underset{ind}{\underline{v}} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$ all wrt ground

$i_1 = g(v_2 - v_3)$, $i_2 = -g(v_1 - v_3)$, $i_3 = -(i_1 + i_2) + \frac{1}{sL}(v_3 - v_4)$

$i_4 = -i_L$ of above $= +\frac{1}{sL}(v_4 - v_3)$

nodal admittance if ground node 4

$$\begin{array}{l} i_1 \rightarrow \\ i_2 \rightarrow \\ i_3 \rightarrow \\ i_4 \rightarrow \end{array} \quad Y_{ind} = \begin{bmatrix} 0 & g & -g & 0 \\ -g & 0 & g & 0 \\ g & -g & \frac{1}{sL} & -\frac{1}{sL} \\ 0 & 0 & -\frac{1}{sL} & \frac{1}{sL} \end{bmatrix}$$

here all entries in a row sum to zero
" " " " a column sum to zero

this is singular. Can make "definite" = a nodal admittance by putting a ground on the circuit i.e. forces one voltage to zero

If $v_4 = 0 \Rightarrow$
$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11\,ind} & y_{12\,ind} \\ \hline y_{21\,ind} & y_{ind\,22} \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ 0 \end{bmatrix}$$

eliminate the current to the ground node (i.e. ignore)
last column is multiplied by 0 so ignore

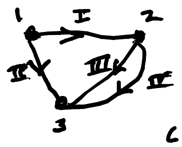
$\therefore\ Y_{nodal} = Y_{11\,ind}$ if scratch out last row & column, i.e. ground last node

Graph theory for finite circuits

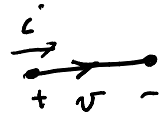
set of points = nodes $\qquad$ # of nodes = n

& lines = branches $\qquad$ # of branches = b

oriented
put arrows on branches
⇒ positive direction of current in a branch

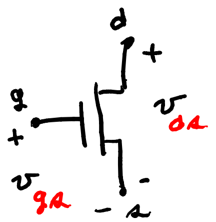


$b = 4$
$n = 3$



⇒ power in = $v \cdot i$ for a branch ⇒ arrow gives polarity for the voltage

Ex:



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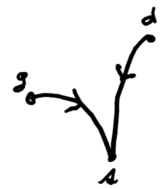

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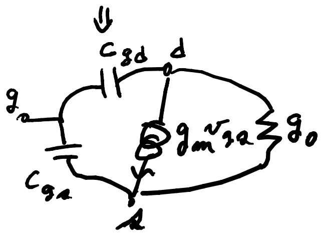

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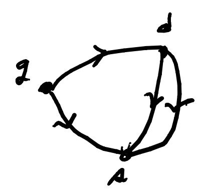
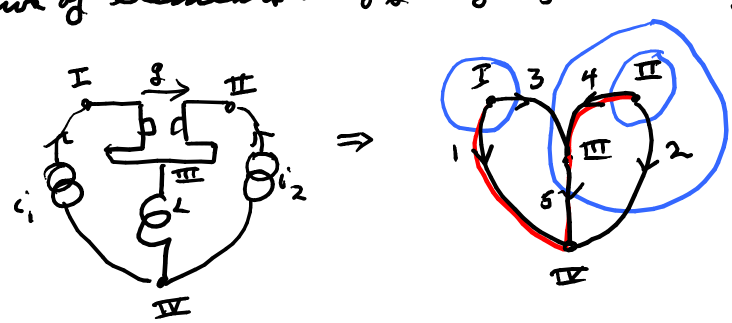


Laws of connection: KCL & KVL

Laws of elements: y's or z's, scattering



$b = 5$
$n = 4$
$t = 3 = n - 1$
$l = 2 = b - t$
$\quad = b - (n-1)$
$\quad = b - n + 1$

tree = set of branches connecting all node with not closed path $\qquad$ # of tree branches = t

cotree = non-tree branches = leaves $\qquad$ # of leaves = l

Cut set equations, C = cut set matrix, KCL on a sphere cutting only one tree branch

for branch 3: $\quad 0 = i_1 + i_3$

4: $\quad 0 = i_2 + i_4$

5: $\quad 0 = i_2 - i_3 \qquad i_5$