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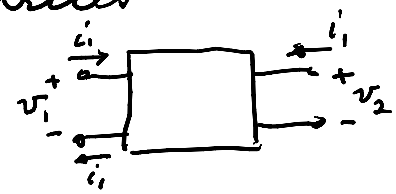
EE610
09/03/09b

admittance matrices

2-port

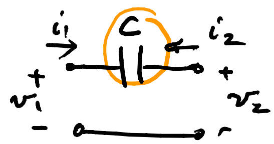


$$V(s) = \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}$$

$$I(s) = \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$Y(s)V(s) = I(s)$

Ex:



$y_{cap} = sC = Cs$

$i_1(t) = C \dfrac{d(v_1 - v_2)}{dt} =$

KCL: $i_1 + i_2 = 0$

$I_1(s) = -I_2(s)$, $I_1(s) = Cs V_1(s) - Cs V_2(s)$

$$= \begin{bmatrix} Cs & -Cs \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}$$

$I_2(s) = -I_1(s)$

$$= \begin{bmatrix} -Cs & +Cs \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Cs & -Cs \\ -Cs & +Cs \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow Y(s) = \begin{bmatrix} Cs & -Cs \\ -Cs & Cs \end{bmatrix} = Cs \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$\det Y(s) = c^2 s^2 - (-cs)^2 = 0$, $\therefore$ no Y^{-1} exists

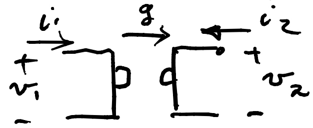
but $Y(s) = Y^T(s)$ = transpose; $Y(s)$ is symmetric

$\therefore$ $V(s) = Z(s) I(s)$ has no 2-port $Z(s)$

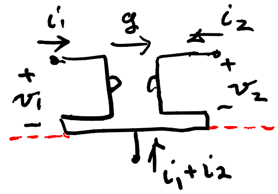
Ex: gyrator: $Y(s) = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$; here $Y^T = -Y \Rightarrow Y + Y^T = 0_{2\times 2}$

$$= \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} = \begin{bmatrix} -0 & -g \\ -(-g) & -0 \end{bmatrix}$$

$\therefore$ $Y(s)$ = skew-symmetric



$\Rightarrow$



Here $\det Y(s) = 0^2 - (g)(-g) = g^2$

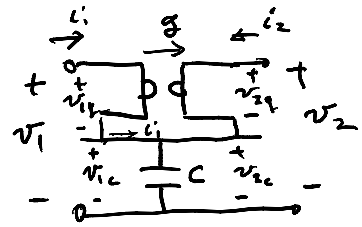
$\therefore$ Y_{gyr} is not singular if $g \neq 0$

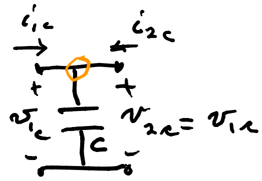
∴ Y^{-1} exists (if $g \neq 0$) & $\Rightarrow Y^{-1}_{gyr} = Z_{gyr} = \frac{1}{g^2}\begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$

r = gyration resistance

also $Z_{gyr} = -Z_{gyr}^T$

Ex:





$I_c = I_{1c} + I_{2c}$
$= sC\, V_{1,2}$

$V = ZI = \begin{bmatrix} \frac{1}{sC} & \frac{1}{sC} \\ \frac{1}{sC} & \frac{1}{sC} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_c = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_c$

$i_{1c} = i_1 = i_{1g}$, $v_1 = v_{1g} + v_{1c}$

$i_{2c} = i_2 = i_{2g}$ $\quad v_2 = v_{2g} + v_{2c}$

$Z_c = \frac{1}{sC}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\det Z_c = 0$

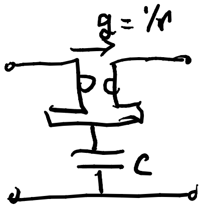
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{1g} \\ V_{2g} \end{bmatrix} + \begin{bmatrix} V_{1c} \\ V_{2c} \end{bmatrix} = Z_{gyr}\begin{bmatrix} I_{1g} \\ I_{2g} \end{bmatrix} + Z_c\begin{bmatrix} I_{1c} \\ I_{2c} \end{bmatrix} = (Z_{gyr} + Z_c)\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z = Z_{gyr} + Z_c = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{sC} & \frac{1}{sC} \\ \frac{1}{sC} & \frac{1}{sC} \end{bmatrix} = \begin{bmatrix} \frac{1}{sC} & r + \frac{1}{sC} \\ -r + \frac{1}{sC} & \frac{1}{sC} \end{bmatrix}$$

given any square matrix A

$$A = \frac{A + A^T}{2} + \frac{A - A^T}{2} = A_{symmetric} + A_{skew}$$

$g = 1/r$



$\Rightarrow Z = \begin{bmatrix} \frac{1}{sC} & r + \frac{1}{sC} \\ -r + \frac{1}{sC} & \frac{1}{sC} \end{bmatrix}$; $\det Z = \left(\frac{1}{sC}\right)^2 - \left(r + \frac{1}{sC}\right)\left(-r + \frac{1}{sC}\right)$
$= \left(\frac{1}{sC}\right)^2 + r^2 - \frac{1}{sC^2} = r^2$

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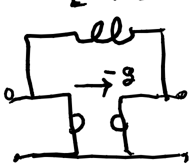
$Y = \frac{1}{r^2}\begin{bmatrix} \frac{1}{sC} & \left(r + \frac{1}{sC}\right)(-1)^3 \\ \left(-r + \frac{1}{sC}\right)(-1)^{2+1} & \frac{1}{sC} \end{bmatrix}$

$L = r^2 C$

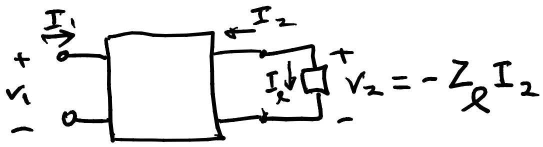


$= g^2\begin{bmatrix} \frac{1}{sC} & -r - \frac{1}{sC} \\ r - \frac{1}{sC} & \frac{1}{sC} \end{bmatrix}$

$= \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{sr^2C} & -\frac{1}{sr^2C} \\ \frac{-1}{sr^2C} & \frac{1}{sr^2C} \end{bmatrix}$

$= Y_{gyr} + Y_{ind}$

Let's load with a short



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} V_1 \\ -Z_\ell I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$-Z_\ell I_2 = z_{21} I + z_{22} I_2 \Rightarrow -(Z_L + z_{22}) I_2 = z_{21} I_1$$

$I_2 = -(Z_\ell + z_{22})^{-1} z_{21} I_1$ into 1st row

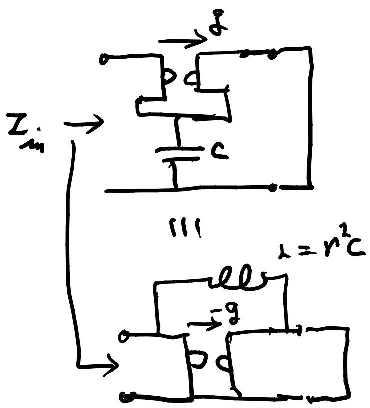
$$V_1 = z_{11} I_1 + z_{12} I_2 = [z_{11} - z_{12}(Z_\ell + z_{22})^{-1} z_{21}] I_1$$

$$= Z_{in} \cdot I_1$$

here $Z_{in} = \dfrac{z_{11} Z_\ell + z_{11} z_{22} - z_{12} z_{21}}{Z_\ell + z_{22}} = \dfrac{z_{11} Z_\ell + \det Z}{Z_\ell + z_{22}}$

for a short $v = 0$, i = anything $\Rightarrow Z_\ell = 0$ if i is finite

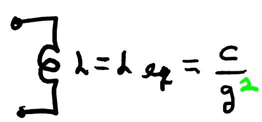
for our circuit, $\det Z = r^2$



$$Z_{in} = \frac{r^2}{\frac{1}{s r^2 C}} = s r^2 C = s L_{eq}.$$

corrected denominator

$\equiv$



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